

LIMITS AND ASYMPTOTES – A SUMMARY

I. EVALUATING $\lim_{x \rightarrow a} f(x)$, where a is finite

A. NON-PIECEWISE DEFINED

Compute $f(a)$

1. If $f(a)$ is a finite number, then this number is the answer.

2. If $f(a) = \frac{0}{0}$, the limit is *indeterminate*.

If possible, simplify $f(x)$ algebraically, reevaluate $f(a)$ and determine the answer from the choices above.

If $f(x)$ cannot be simplified algebraically, use the calculator to try to guess the answer.

3. If $f(a) = \frac{K}{0}$ where $K \neq 0$, consider 1-sided limits to determine whether the denominator $\rightarrow 0^-$ or $\rightarrow 0^+$, and whether the numerator is positive or negative.

If the 1-sided limits have the same signs, the limit does not exist, but it does not exist because it is becoming infinite. You must decide if the answer will be $+\infty$ or $-\infty$.

If the 1-sided limits do not have the same signs, you can only conclude that the limit does not exist.

VERTICAL ASYMPTOTES

If any of the limits $\lim_{x \rightarrow a} f(x)$, $\lim_{x \rightarrow a^-} f(x)$, or $\lim_{x \rightarrow a^+} f(x)$ is infinity or negative infinity, then the line $x = a$ is a vertical asymptote.

B. PIECEWISE DEFINED

1. If a is not a boundary value, use the rules for non-piecewise functions.

2. If a is a boundary value, consider 1-sided limits. If 1-sided limits are not equal, the limit does not exist. If 1-sided limits are equal, the limit has the same value as the 1-sided limits.

II. EVALUATING $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)}$ or $\lim_{x \rightarrow -\infty} \frac{f(x)}{g(x)}$, where $f(x)$ and $g(x)$ are algebraic

If $f(x)$ and $g(x)$ both $\rightarrow +\infty$ or $-\infty$, then the limit is indeterminate.

OVER \rightarrow

A. FORMAL APPROACH

Divide the entire numerator and the entire denominator by the term with the highest power in the denominator. Evaluate the limit, using the fact that

$\lim_{x \rightarrow \infty} \frac{k}{x^n} = 0$ and $\lim_{x \rightarrow \infty} \frac{k}{x^n} = 0$, where k is a non-zero constant and n is a positive constant.

B. INFORMAL APPROACH

Consider only the term with the highest power in the numerator and the term with the highest power in the denominator. Simplify and reevaluate the limit. If your answer is infinite think carefully about whether it should be $+\infty$ or $-\infty$.

C. HORIZONTAL ASYMPTOTES

Using either the formal or the informal approach, if $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = c$ or $\lim_{x \rightarrow -\infty} \frac{f(x)}{g(x)} = c$,

where c is a finite constant, then the line $y = c$ is a horizontal asymptote.