MA 181 Test #1 – Spring, 2010

Riseberg Name Solutions

Show all of your work on the test paper. Full credit is not given unless the answer follows from the work shown.

1. (8 points) Classify each function as a power function, polynomial function, rational function, or exponential function.

(a) $f(x)=\left(\frac{1}{2}\right)^x$ exponential

(b) $f(x)=x^{1/2}$ power

(c) $f(x)=\frac{2x+1}{x^2-4}$ rational

(d) $f(x)=\frac{2}{3}x^4-3x^2+7.2$ polynomial

2. (10 points) When exercising, you should raise your pulse rate to your target pulse rate. Your target pulse rate depends on your age (see the table to the right). Let $y=f(x)$ represent the target pulse rate (in beats per minute) for a person who is x years old.

Age	Target Pulse Rate (beats per minute)
20	150
40	135
60	120

Note that the relationship between x and y appears to be linear.

(a) ***Without using the STAT feature of your calculator,*** find the slope and equation of the linear function through the given data. Be sure to show your work.

$m=-\frac{15}{20}=-.75$

$y-150=-.75(x-20)$

$y=-.75x+165$

(b) Write a sentence explaining what the slope of the function means in this situation. Use everyday language, and be sure to use appropriate units.

The slope of -.75 means that the target pulse rate decreases by ,75 beats per minute for each additional year of age.

3. (8 points) If a ball is projected vertically upward from the surface of the moon with a velocity of 64 ft/s, its height in feet after t seconds is given by $h(t)=-2.6t^2+64t$.

(a) Find the average velocity of the ball during each of the following time intervals. **Use appropriate units in your answers.** Write your answers correct to at least two decimal places.

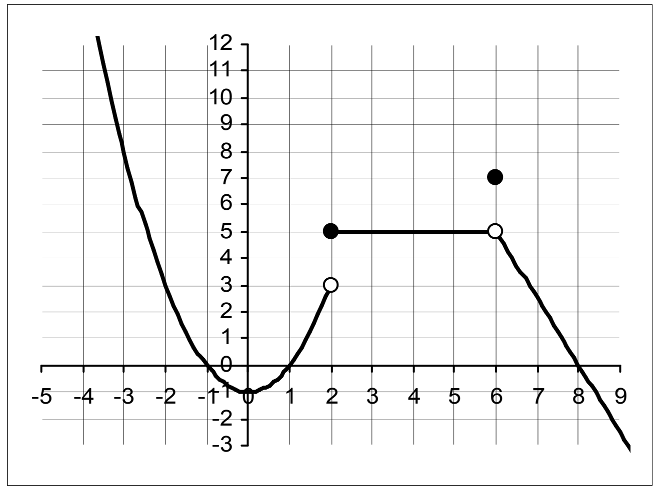
(i) [5, 5.1] $\frac{h(5.1)-h(5)}{5.1-5}=\frac{3.774}{.1}=37.74\ ft/\sec$

(ii) [5, 5.01] $\frac{h(5.01)-h(5)}{5.01-5}=\frac{.37974}{.01}=37.974\ ft/\sec$

(b) Based on the results of part (a), what is the instantaneous velocity of the ball at t = 5 seconds?

38 ft/sec

4. (16 points) The function f is defined by the graph shown. Using information from this graph, determine each of the following, if possible. If a limit does not exist, state this.



(i) $\lim_{x \to 2^-} f(x) = 3$

(ii) $\lim_{x \to 2^+} f(x) = 5$

(iii) $\lim_{x \to 2} f(x)$ does not exist

(iv) $f(2) = 5$

(v) $\lim_{x \to 6^-} f(x) = 5$

(vi) $\lim_{x \to 6^+} f(x) = 5$

(vii) $\lim_{x \to 6} f(x) = 5$

(viii) $f(6) = 7$

5. (7 points) ***Algebraically*** (without using your calculator), determine the following limit. Express your answer as an integer or fraction, not as a decimal number. If the limit does not exist, state this.

$$\lim_{x \to 3} \frac{x^2 - 4x + 3}{x^2 + 2x - 15} = \lim_{x \to 3} \frac{(x-3)(x-1)}{(x-3)(x+5)} = \lim_{x \to 3} \frac{(x-1)}{(x+5)} = \frac{2}{8} = \frac{1}{4}$$

6. (8 points) Determine the value of k so that the function f(x) given below is continuous for all x.

$$f(x) = \begin{cases} 4x + 3 & if \quad x \le k \\ 5 - 3x & if \quad x > k \end{cases}$$

$$\lim_{x \to k^-} f(x) = \lim_{x \to k^-} (4x + 3) = 4k + 3$$

$$\lim_{x \to k^+} f(x) = \lim_{x \to k^+} (5 - 3x) = 5 - 3k$$

For the function to be continuous, the left and right limits must be equal, so

$$4k + 3 = 5 - 3k$$

$$7k = 2$$

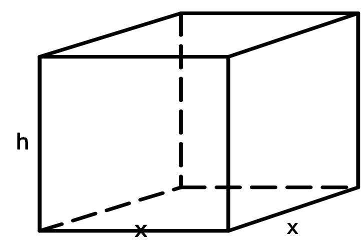
$$k = \frac{2}{7}$$

7. (6 points) Let $f(x) = x^3 - 3x^2 + 7x - 9$. Does the Intermediate Value Theorem guarantee that this function has a zero in the interval (1, 2)? Why or why not?

$f(1) = -4 \text{ and } f(2) = 1$

The function is continuous everywhere since polynomials are continuous for all x. The Intermediate Value Theorem guarantees that the function takes on every value between -4 and 1 for x in the interval (1, 2). Since zero is between -4 and 1, there is a number c between 1 and 2 such that f(c) = 0, that is, c is a zero of f(x).

8. (8 points) A closed box has a square base. If the volume of the box is 12 cubic feet, express the total surface area of the box as a function of the length of a side of the base, **x,** and **simplify your answer.**



$$V = x^2 h = 12 \quad so \quad h = \frac{12}{x^2}$$

$$S = 2x^2 + 4xh = 2x^2 + 4x\left(\frac{12}{x^2}\right) = 2x^2 + \frac{48}{x}$$

9. (4 points) Evaluate the limit. Express your answer as ∞, $-\infty$, an integer, or a fraction, not as a decimal number.

$$\lim_{x\to 3^+} \frac{x-5}{x-3} \Rightarrow \frac{-2}{0^+} \Rightarrow -\infty$$

10. (10 points) Given the function $f(x) = \frac{x-2}{x^2 - 8x + 15}$,

(a) State the equation(s) of any vertical asymptotes of $f(x)$.

$$f(x) = \frac{x-2}{x^2 - 8x + 15} = \frac{x-2}{(x-3)(x-5)}$$

The vertical asymptotes are x = 3 and x = 5.

(b) State the equation(s) of any horizontal asymptotes of $f(x)$.

$$\lim_{x\to\infty} \frac{x-2}{x^2 - 8x + 15} = \lim_{x\to\infty} \frac{\frac{x}{x^2} - \frac{2}{x^2}}{\frac{x^2}{x^2} - \frac{8}{x^2} + \frac{15}{x^2}} = \frac{0}{1} = 0$$

The horizontal asymptote is y = 0.

11. (15 points) A function f is discontinuous at x = 2 and x = 5 but is continuous elsewhere. In addition, f satisfies **all** of the following conditions:

$\lim_{x \to 5^-} f(x) = \infty$ $\lim_{x \to 5^+} f(x) = -\infty$ $\lim_{x \to 2} f(x) = -\infty$

$\lim_{x \to \infty} f(x) = -3$ $\lim_{x \to -\infty} f(x) = -3$

Sketch a possible graph for f. Be sure to show all vertical and horizontal asymptotes.