MA 181 Test #2 – Spring, 2010

Riseberg (100 points) Name Solutions

1. (8 points) Use the definition of the derivative to determine $f'(x)$ if $f(x)=x^2-3x+5$.
No credit will be given if any other method is used.

$$f'(x)=\lim_{h\to 0}\frac{(x+h)^2-3(x+h)+5-(x^2-3x+5)}{h}$$
$$=\lim_{h\to 0}\frac{x^2+2xh+h^2-3x-3h+5-x^2+3x-5}{h}$$
$$=\lim_{h\to 0}\frac{2xh+h^2-3h}{h}=\lim_{h\to 0}\frac{h(2x+h-3)}{h}$$
$$=\lim_{h\to 0}(2x+h-3)=2x-3$$

2. (6 points) Let $P(t)$ be the population of the United States in millions t years after 1900.
Write a sentence explaining the meaning of each of the following in everyday language.
Use appropriate units in your answers.

(a) $P(80)=226.5$

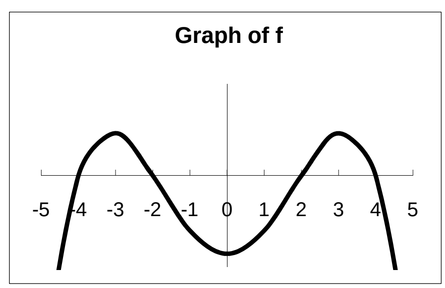
In 1980, the U.S. population was 226.5 million people.

(b) $P'(80)=2.2$

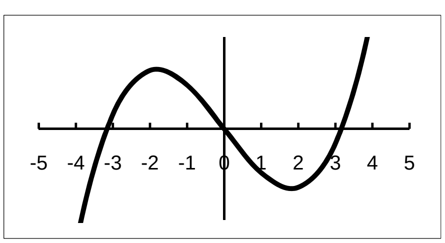
In 1980, the U.S. population was growing at a rate of 2.2 million people per year.

3. (4 points) The graph of the function f is shown to the right.

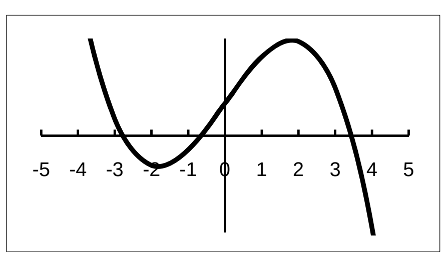
Circle the graph below which could be the graph of f', the derivative of f.



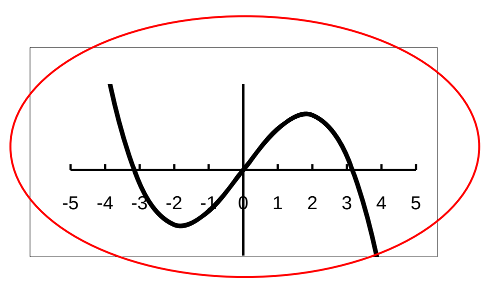
Graph A



Graph B



Graph C



4. (8 points) State whether each statement is true or false.

(a) If a function f is continuous at x = a, then it must be differentiable at x = a. False

(b) If a function f is differentiable at x = a, then it must be continuous at x = a. True

(c) The function $g(x) = \frac{1}{x}$ has as its derivative $g'(x) = -\frac{1}{x^2}$. Therefore, the equation of the tangent line to g(x) at the point (2, ½) is $y - \frac{1}{2} = -\frac{1}{x^2}(x-2)$. False

(d) The function $f(x) = |x|$ is differentiable for all x. False

5. (14 points) Suppose that the function *f* is differentiable for all x and that it satisfies all of the following conditions:

$f(0) = 0$

$f'(3) = f'(10) = 0$

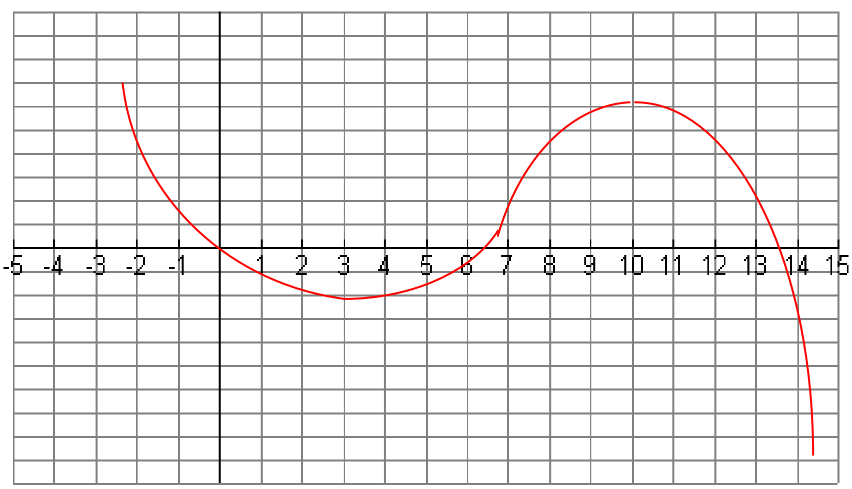
$f'(x) < 0 \text{ on } (-\infty, 3) \text{ and } (10, \infty)$

$f'(x) > 0 \text{ on } (3, 10)$

$f''(x) > 0 \text{ on } (-\infty, 7)$

$f''(x) < 0 \text{ on } (7, \infty)$

(a) Sketch a possible graph of *f*.



(b) At what value or values of x does *f* have a

(i) local minimum x = 3

(ii) local maximum x = 10

(iii) point of inflection x = 7

6. (20 points) Find the derivative of each function. Answers should be left with no negative exponents.

(a)

$$f(x) = \frac{x^3}{5} + \frac{5}{x^3} = \frac{1}{5}x^3 + 5x^{-3}$$

$$f'(x) = \frac{3}{5}x^2 - 15x^{-4} = \frac{3}{5}x^2 - \frac{15}{x^4}$$

(b)

$$g(x) = x\sqrt{x} + \sqrt[5]{x^2} = x^{3/2} + x^{2/5}$$

$$g'(x) = \frac{3}{2}x^{1/2} + \frac{2}{5}x^{-3/5} = \frac{3}{2}x^{1/2} + \frac{2}{5x^{3/5}}$$

(c)

$$y = x^2 e^x$$

$$y' = x^2 e^x + e^x 2x = xe^2(x+2)$$

7. (10 points) Find $f'(x)$ and simplify your answer.

$$f(x) = \frac{x^3 + 4}{2x - 7}$$

$$f'(x) = \frac{(2x-7)3x^2 - (x^3+4)2}{(2x-7)^2} = \frac{6x^3 - 21x^2 - 2x^3 - 8}{(2x-7)^2} = \frac{4x^3 - 21x^2 - 8}{(2x-7)^2}$$

8. (10 points) Find an equation of the tangent line to the function $f(x) = 2\sin x + 3\cos x$ when x = 0 on the curve.

$$f(x) = 2\sin x + 3\cos x$$

$$f'(x) = 2\cos x - 3\sin x$$

$$x = 0$$

$$y = f(0) = 2\sin 0 + 3\cos 0 = 3$$

$$m = f'(0) = 2\cos 0 - 3\sin 0 = 2$$

$$y - 3 = 2(x - 0) \Rightarrow y = 2x + 3$$

9. (10 points) For what value or values of x does the function $f(x)=\frac{1}{3}x^3-x^2-8x+5$ have a horizontal tangent line?

$$f'(x)=x^2-2x-8=0$$

$$(x-4)(x+2)=0$$

$$x-4=0 \quad x+2=0$$

$$x=4 \quad x=-2$$

10. (10 points) Find a parabola $f(x)=ax^2+bx+c$ that satisfies the following conditions:

- $f(x)$ passes through the point (0, 6)
- $f'(1)=7$ and $f'(-1)=9$

$$f(x)=ax^2+bx+c$$

$$f(0)=c=6$$

$$f'(x)=2ax+b$$

$$f'(1)=2a+b=7$$

$$f'(-1)=-2a+b=9$$

$$Adding: 2b=16 \Rightarrow b=8$$

$$2a+8=7 \Rightarrow 2a=-1 \Rightarrow a=-\frac{1}{2}$$

$$f(x)=-\frac{1}{2}x^2+8x+6$$