MA 181 Finding a Formula for the Surface Area of a Box
Section 1.1

Later in the semester, we will use the methods of Calculus to solve problems involving ***optimization****;* that is, finding the ***best*** way of doing something. Many of these problems involve geometric figures in which it is necessary to express one quantity as a function of another. Here is an example of such a problem.

A large rectangular box, open at the top, is to be constructed so that the volume is 20 cubic feet and the base has length equal to twice its width. We will seek a formula for the *surface area* of the box as a function of the width.

(1) First draw a picture of the box and label its dimensions. Use h for the height, w for the width, and write the length in terms of w.

(2) Write an expression in terms of h and w for the surface area S of the box. To do this, you will have to add up the areas of the bottom and the four sides of the box.

(3) Now consider the volume of the box. You are given that it is 20 ft^3. Use this fact to write an equation relating h and w and then solve for h in terms of w.

(4) Substitute the expression for h which you found in (3) into the expression for S in (2) and simplify the result. You should now have an expression ***for surface area as a function of w***. What do you think is a reasonable domain for this function?

(5) Use your calculator to graph the function that you found for S. Be sure to set a viewing window on your calculator that makes sense for the function which you are graphing.

(6) Based on your graph, is it possible to construct a box with the given specifications such that the surface area is 80 ft^2? 50 ft^2? 25 ft^2? Why or why not?

(7) From your graph, determine the approximate value of x which will give the ***minimum*** value for the surface area of the box and what this minimum surface area will be. Why do you think minimizing the surface area might be important?