For each rational function f(x) below,

(a) Divide the entire numerator and the entire denominator by the highest power of x which is present in the denominator. Separate the numerator and denominator into individual terms and simplify each term. Then determine $\lim_{x\to\infty} f(x)$ by looking at what happens to each term as $x \to \infty$. Keep in mind that if n is a positive integer and c is any real number, $\lim_{x\to\infty} \frac{c}{x^n} = 0$. By using this process, you are determining $\lim_{x\to\infty} f(x)$ ***algebraically***.

(b) In order to determine $\lim_{x\to -\infty} f(x)$ algebraically, you can use the same process as in (a) and keep in mind that that if n is positive and c is any real number, $\lim_{x\to -\infty} \frac{c}{x^n} = 0$.

(c) ***Numerically*** confirm your answers to parts (a) and (b). To do this, use the table feature on your calculator with increasingly large values of x for $x \to \infty$ (for example, x = 10, 100, 1000 and so on). For $x \to -\infty$, use values of x which are negative but which get increasingly large in absolute value (for example, x = -10, -100, -1000 and so on).

(d) Use your calculator to graph the function. Make sure you choose window settings which will give a complete graph of the function. Does the function have a horizontal asymptote? If so, what is its equation?

(e) Based on your graph and your answers to parts (a) - (d), under what circumstances will a rational function have a horizontal asymptote? How can you tell just by looking at the function whether it has a horizontal asymptote and, if so, what the equation of the horizontal asymptote is?

1. $f(x) = \dfrac{6x^2 - 3x - 1}{2x^2 + 8}$

2. $f(x) = \dfrac{6x + 1}{2x^2 + 8}$

3. $f(x) = \dfrac{6x^3 - 3x - 1}{2x^2 + 8}$

4. $f(x) = \dfrac{6x^4 - 3x - 1}{2x^2 + 8}$