

Derivatives of Trigonometric Functions

1. You are going to try to guess the derivative of $y = \sin x$ by looking at the graph of a difference quotient.
 - (a) Enter the difference quotient $\frac{\sin(x + .0001) - \sin x}{.0001}$ as y_1 on your calculator, and graph y_1 on a window of $[-2\pi, 2\pi]$ for x and $[-2, 2]$ for y . What does the graph look like?
 - (b) Change the difference quotient to $\frac{\sin(x - .0001) - \sin x}{-.0001}$ and graph y_1 again. What does this graph look like? Is it any different from your answer to part (a)?
 - (c) What can you conclude about $\frac{d}{dx}(\sin x)$?
2. Redo problem #1 for $g(x) = \cos x$. What can you conclude about $\frac{d}{dx}(\cos x)$?
3. Let $y = \tan x = \frac{\sin x}{\cos x}$. Find $y' = \frac{d(\tan x)}{dx}$ by applying the quotient rule. Your answer will contain a fraction. Rewrite the answer using an appropriate reciprocal function so there are no fractions in your answer.
4. Similarly, let $y = \sec x = \frac{1}{\cos x}$. Find $y' = \frac{d(\sec x)}{dx}$ by applying the quotient rule. Once again, your answer will contain a fraction. Rewrite it using appropriate reciprocal functions so there are no fractions in your answer.
5. Based on your answers to 1 - 4 above, fill in the differentiation formulas below:

$$\frac{d}{dx}(\sin x) =$$

$$\frac{d}{dx}(\tan x) =$$

$$\frac{d}{dx}(\cos x) =$$

$$\frac{d}{dx}(\sec x) =$$

Exercises

1. Find the derivative for each of the following functions.
 - (a) $y = 2\sin x + 3\cos x$
 - (b) $y = x^2 \sin x$

(c) $y = \frac{\tan x}{e^x}$

(d) $y = \sec x \tan x$

(e) $y = \frac{\cos x}{1 + \sin x}$

2. (a) Find the equation of the tangent line to $y = x \cos x$ when $x = \pi$ on the curve.

(b) Illustrate part (a) by graphing the curve and the tangent line on the same screen.

3. **Analytically** determine the exact values of all x in $[0, 2\pi]$ such that the graph of $f(x) = x + 2\sin x$ has a horizontal tangent line. Check your answers by determining the approximate values **graphically**.

Answers:

1. (a) $2 \cos x - 3 \sin x$
(b) $2x \sin x + x^2 \cos x$

(c) $\frac{\sec^2 x - \tan x}{e^x}$

(d) $\sec x (\sec^2 x + \tan^2 x)$

(e) $\frac{-\sin x - \sin^2 x - \cos^2 x}{(1 + \sin x)^2} = \frac{-1}{1 + \sin x}$

2. (a) $y = -x$

3. $x = 2\pi/3, 4\pi/3$