MA 181 Derivatives of Logarithmic Functions
Section 3.7

1. Consider the function $y = \log_a x$. So far, we have not found a formula for the derivative of this function. To find such a formula,

(a) Recall that the inverse of a logarithmic function is an exponential function. Rewrite $y = \log_a x$ in exponential form as $x = a^y$ and use implicit differentiation to find $\frac{dy}{dx}$. Write your answer so that it is in terms of x only (not y).

(b) Now use this formula to find $\frac{d(\ln x)}{dx}$. (Recall that $\ln x$ means $\log_e x$.)

2. Use the formulas that you developed above to differentiate each of the following functions.

(a) $y = \frac{\ln x}{x^2}$ (b) $y = \sin(\ln x)$ (c) $y = \log_5 x$

3. Use the Chain Rule to extend the formulas from #1 above to determine

(a) $\frac{d[\ln(g(x))]}{dx}$ (b) $\frac{d[\log_a(g(x))]}{dx}$

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4. Use the formulas from #3 to differentiate the following functions.

(a) $y = \ln(x^3 + 2)$ (b) $y = \ln(\sin x)$ (c) $y = \log_3(5x + 2)$

5. There are three rules of logarithms which can be used to rewrite expressions involving logarithms so that these expressions do not contain products, quotients, or extended power functions. You should already be familiar with these rules. They are:

Base a Logarithms	Natural Logarithms
(1) $\log_a(MN) = \log_a M + \log_a N$	(1) $\ln(MN) = \ln M + \ln N$
(2) $\log_a \frac{M}{N} = \log_a M - \log_a N$	(2) $\ln \frac{M}{N} = \ln M - \ln N$
(3) $\log_a M^r = r \log_a M$	(3) $\ln M^r = r \ln M$

Use one or more of these rules, as appropriate, to rewrite each of the following expressions **before differentiating** and then find the derivative.

(a) $y = \ln(3x^4 + 5x)^7$

(b) $y = \ln \frac{3x + 1}{\sqrt{x^2 + 2}}$

Answers for exercises:

2. (a) $\frac{1 - 2\ln x}{x^3}$ (b) $\frac{\cos(\ln x)}{x}$ (c) $\frac{1}{x \ln 5}$

4. (a) $\frac{3x^2}{x^3 + 2}$ (b) $\frac{\cos x}{\sin x} = \cot x$ (c) $\frac{5}{\ln 3(5x + 2)}$

5. (a) $\frac{7(12x^3 + 5)}{3x^4 + 5x}$ (b) $\frac{3}{3x + 1} - \frac{x}{x^2 + 2}$