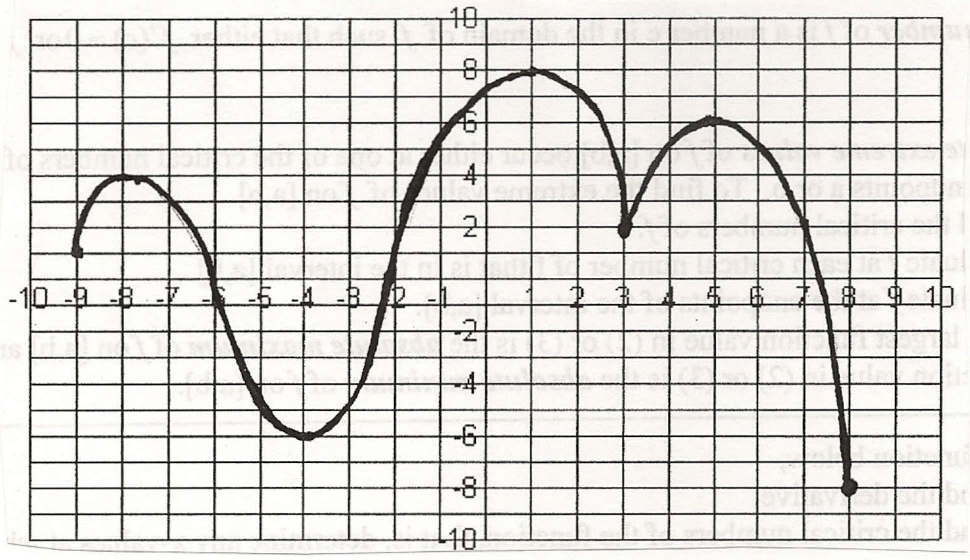
MA 181
Section 4.2

Maximum and Minimum Values



Definitions

- A function has an ***absolute (or global) maximum*** at x = c if f(c) ≥ f(x) for all x in the domain of f. f(c) is called the maximum value of f on its domain.
- A function has an ***absolute (or global) minimum*** at x = c if f(c) ≤ f(x) for all x in the domain of f. f(c) is called the minimum value of f on its domain.

The absolute maximum and minimum values are called the absolute extreme values of the function. Essentially, they are the highest and lowest y-values of the function respectively.

1. Refer to the function f graphed above. At what value of x does f have its absolute maximum value? What is the maximum value of the function?

2. At what value of x does f have its absolute minimum value? What is the minimum value of the function?

Definitions

- A function has a ***local (or relative) maximum*** at x = c if f(c) ≥ f(x) for all x in an open interval containing x = c.
- A function has a ***local (or relative) minimum*** at x = c if f(c) ≤ f(x) for all x in an open interval containing x = c.

3. Refer again to the function in the graph above. At what value(s) of x does f have a local maximum?

4. At what value(s) of x does f have a local minimum?

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The Extreme Value Theorem If f is continuous on a closed interval [a,b], then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in [a,b].

A critical number of f is a number c in the domain of f such that either $f'(c)=0$ or $f'(c)$ does not exist.

The ***absolute extreme values*** of f on [a,b] occur either at one of the critical numbers of f in [a,b] or at one of the endpoints a or b. To find the extreme values of f on [a,b]

(1) Find the critical numbers of f.
(2) Evaluate f at each critical number of f that is in the interval [a,b].
(3) Evaluate f at the endpoints of the interval [a,b].
(4) The largest function value in (2) or (3) is the ***absolute maximum*** of f on [a,b] and the smallest function value in (2) or (3) is the ***absolute minimum*** of f on [a.b].

For each function below,

(a) Find the derivative.
(b) Find the critical numbers of the function, that is, determine any x-values at which the derivative is either zero or undefined.
(c) Determine the absolute maximum and the absolute minimum of the function on the interval [-4,4]. If the function does not have an absolute maximum or an absolute minimum on [-4,4], state this and explain why this does not contradict the Extreme Value Theorem.

5. $f(x)=x^3+3x^2-9x-2$	6. $m(x)=\dfrac{x}{x^2+9}$
7. $f(x)=(x^2-9)^{2/3}$	8. $h(x)=\dfrac{1}{(x^2-9)^{1/3}}$