MA 181 Critical Numbers & Extreme Values on a Closed Interval
Section 4.2

The Extreme Value Theorem If f is continuous on a closed interval [a,b], then f attains an absolute maximum value f(c) and an absolute minimum value f(d) at some numbers c and d in [a,b].

The ***absolute extreme values*** of f on [a,b] occur either at one of the critical numbers of f in [a,b] or at one of the endpoints a or b. To find the extreme values of f on [a,b]

(1) Find the critical numbers of f.
(2) Evaluate f at each critical number of f that is in the interval [a,b].
(3) Evaluate f at the endpoints of the interval [a,b].
(4) The largest function value in (2) or (3) is the ***absolute maximum*** of f on [a,b] and the smallest function value in (2) or (3) is the ***absolute minimum*** of f on [a.b].

Consider the functions

1. $f(x) = x^3 + 3x^2 - 9x - 2$

2. $m(x) = \dfrac{x}{x^2 + 9}$

3. $f(x) = (x^2 - 9)^{2/3}$

4. $h(x) = \dfrac{1}{(x^2 - 9)^{1/3}}$

For each function,

(a) Find the derivative.

(b) Determine any x-values at which the derivative is zero.

(c) Determine any x-values at which the derivative is undefined.

(d) List the critical numbers of the function.

(e) Determine the absolute maximum and the absolute minimum of the function on the interval [-4,4]. If the function does not have an absolute maximum or an absolute minimum on [-4,4], state this and explain why this does not contradict the Extreme Value Theorem.