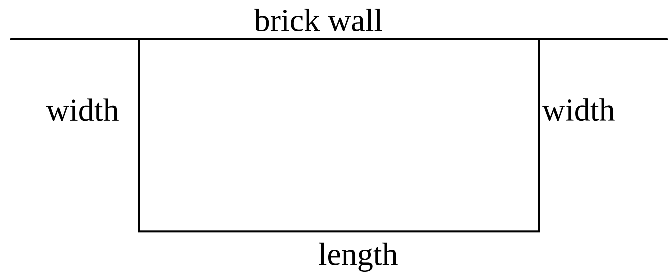
MA 181 Optimization Problems
Section 4.6

1. A farmer has 2500 ft of fencing and wants to fence off a rectangular field that borders a long brick wall. He needs no fencing along the wall. We are going to try to determine the dimensions of the field that will have the largest area.

(a) In order to get a feel for the problem, we'll first fill out a chart showing some possibilities. Keep in mind that the total amount of fencing is 2500 ft.

Width	Length	Area of field
100 ft		
200 ft		
400 ft		
500 ft		
900 ft		



Note that different widths and lengths result in different areas.

(b) Now suppose that the width is x. Write an expression for the length. You will have to use the fact that the total amount of fencing is 2500 ft.

(c) Write an expression for A(x), the area of the rectangular field.

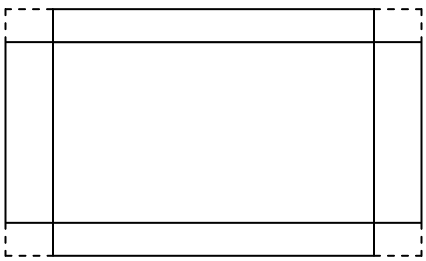
(d) The width of a field must be positive, so we must have $x > 0$. The length also must be positive. Use your expression for y to get another restriction on the value of x. These two inequalities give the domain of the area function.

(e) Use the techniques of Calculus to find the dimensions of the field that will result in a maximum value of the function A on the interval determined by your answer to part (e). What is the maximum value of A?

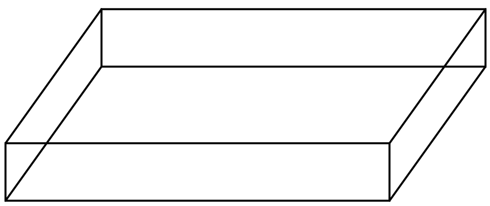
2. At the last minute, you have decided to go trick or treating for Halloween candy. Unfortunately, you don't have a bag, box or plastic pumpkin to put your candy in. All you have is a piece of cardboard that is 10 in by 14 in, a pair of scissors, and tape. You decide to make a box out of the cardboard by cutting out squares from each corner, turning up the sides and taping the sides to make a box. Naturally, you want the box to hold as much candy as possible, so you want the volume of the box to be a maximum. What size square should be cut out of each corner to maximize the volume of the box?

Here's a picture to show what's happening:

Flat piece of cardboard with corners cut out



Cardboard folded up to make a box



(a) Let x be the length of each side of the square that is cut out from each corner. This will become the height of the box.

Write expressions for the length and the width of the box.

(b) The volume of a box is (length)(width)(height.). Using the dimensions you found above, write an expression for the volume V(x) of the box as a function of x.

(c) Since each dimension of the box must be positive, use this to write the domain of V(x).

(d) Use the techniques of Calculus to find the dimensions of the box that will result in a maximum value of the volume function V(x) on the interval determined by your answer to part (c). What is the maximum volume?

3. A rectangular box open at the top is to be constructed so that the volume is 10 cubic meters and the base has length equal to twice its width. Material for the base of the box costs $4 per square meter, while the material for the sides costs $2 per square meter. Our goal is to find the dimensions of the box which will minimize the cost of the box.

To help you in solving this problem, follow these steps.

(a) Draw a picture and label the dimensions of the box. Use h for the height, w for the width and write the length in terms of w.

(b) Write an expression in terms of h and w for the cost C of the box. To do this, you need to recognize that the cost of each surface of the box is the area of that surface multiplied by the cost per square meter. Keep in mind that the box has five surfaces --- the bottom plus four sides.

(c) Use the fact that the volume of the box is 10 m^3 to relate h and w and then solve for h in terms of w.

(d) Substitute the expression for h which you found in (c) into the expression for cost in (b).

(e) You should now have an expression for cost in terms of w only. Use calculus to find the dimensions of the box which will give a minimum for C analytically, find the critical value(s) of C and then use either the First or Second Derivative Test to determine where the function actually has the desired minimum.

(f) Confirm the answer which you got in part (e) by graphing the cost function on your calculator and estimating the value of w which minimizes the cost.

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4. Points A and B are opposite one another on shores of a straight river 3 km wide. Point C is on the same shore as B, but 2 km down the river from B. A telephone company wishes to lay a cable from A to C. Cable laid underwater costs 5 times as much as cable laid on land.

(a) Draw and label a sketch.

(b) If it costs **k** $/km to lay cable on land, how much (in terms of **k**) does it cost to lay cable under water?

(c) How much would it cost to lay cable directly from A to C? (Your answer will be a multiple of **k**.)

(d) What would be the total cost to lay cable from A to B under water, then from B to C on land?

(e) Suppose point D is located on land exactly midway between points B and C. What would be the total cost to lay the cable from A to D under water, then from D to C on land?

(f) The results so far should suggest that the cable paths described in parts (c) and (d) above are not the most cost-efficient. The path described in part (e) might be best, but perhaps there is an *optimum* location for point D, not necessarily midway between B and C.

Let point D be located on a line from B to C, and let x be the distance from B to D. Find expressions for the distance from D to C, and for the distance from A to D.

Write an equation for the total cost function.

(g) Use calculus to minimize the total cost function analytically.

(h) Confirm your answer to part (g) by graphing the cost function on your calculator (assume k = 1) and estimating the value of x (and hence the location of point D) that minimizes the cost.