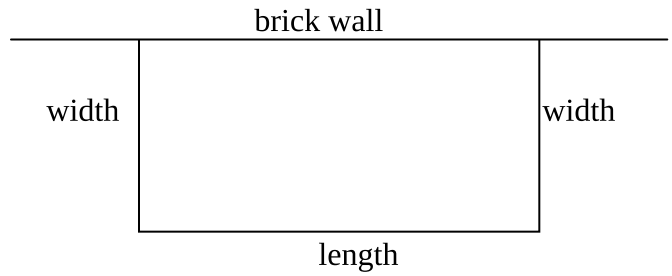
MA 181 Finding a Maximum
Section 4.6

1. A farmer has 2500 ft of fencing and wants to fence off a rectangular field that borders a long brick wall. He needs no fencing along the wall. We are going to try to determine the dimensions of the field that will have the largest area.

(a) In order to get a feel for the problem, we'll first fill out a chart showing some possibilities. Keep in mind that the total amount of fencing is 2500 ft.

Width	Length	Area of field
100 ft		
200 ft		
400 ft		
500 ft		
900 ft		



Note that different widths and lengths result in different areas.

(b) Now suppose that the width is x. Write an expression for the length. You will have to use the fact that the total amount of fencing is 2500 ft.

(c) Write an expression for A(x), the area of the rectangular field.

(d) The width of a field cannot be negative, so we must have $x \geq 0$. The length also cannot be negative. Use your expression for y to get another restriction on the value of x. These two inequalities give the domain of the area function.

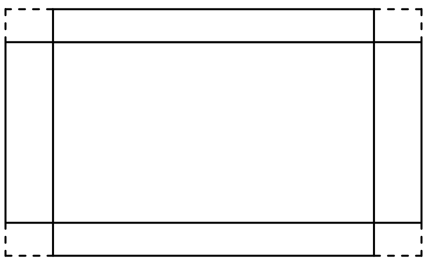
(e) Use the techniques of Calculus to find the dimensions of the field that will result in a maximum value of the function A on the interval determined by your answer to part (e). What is the maximum value of A?

OVER →

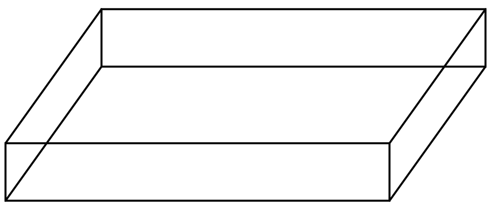
2. At the last minute, you have decided to go trick or treating for Halloween candy. Unfortunately, you don't have a bag, box or plastic pumpkin to put your candy in. All you have is a piece of cardboard that is 10 in by 14 in, a pair of scissors, and tape. You decide to make a box out of the cardboard by cutting out squares from each corner, turning up the sides and taping the sides to make a box. Naturally, you want the box to hold as much candy as possible, so you want the volume of the box to be a maximum. What size square should be cut out of each corner to maximize the volume of the box?

Here's a picture to show what's happening:

Flat piece of cardboard with corners cut out



Cardboard folded up to make a box



(a) Let x be the length of each side of the square that is cut out from each corner. This will become the height of the box.

Write expressions for the length and the width of the box.

(b) The volume of a box is (length)(width)(height.). Using the dimensions you found above, write an expression for the volume V(x) of the box as a function of x.

(c) Since each dimension of the box must be positive, use this to write the domain of V(x).

(d) Use the techniques of Calculus to find the dimensions of the box that will result in a maximum value of the volume function V(x) on the interval determined by your answer to part (c). What is the maximum volume?