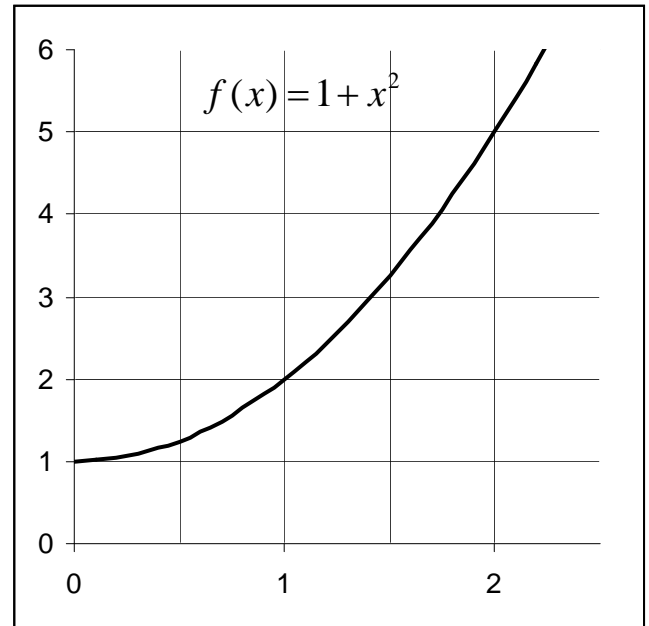
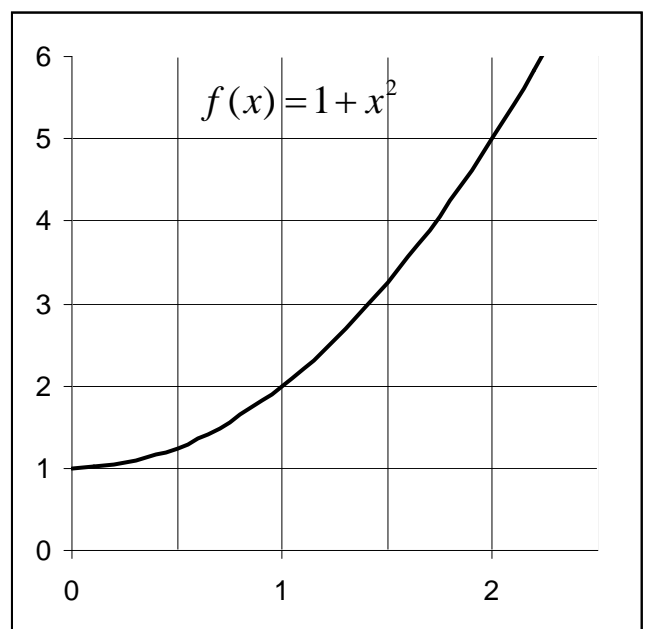


I. Area Under a Curve

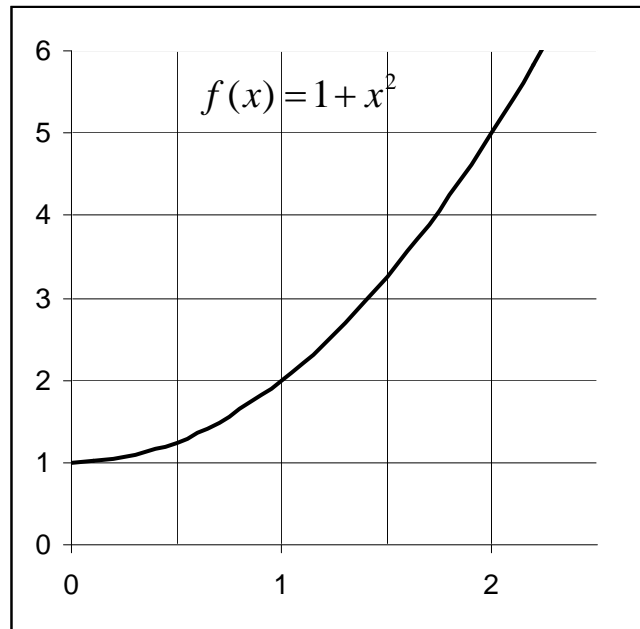
1. Use 4 sub-intervals with sample points at left endpoints to estimate the area under the graph of  $f(x) = 1 + x^2$  on the interval  $[0, 2]$ .



2. Use 4 sub-intervals with sample points at right endpoints to estimate the area under the graph of  $f(x) = 1 + x^2$  on the interval  $[0, 2]$ .



3. Use 4 sub-intervals with sample points at midpoints to estimate the area under the graph of  $f(x) = 1 + x^2$  on the interval  $[0, 2]$ .



II. A Distance Computation

4. An object travels with the **increasing velocity** given in the chart below. Our goal is to estimate the total distance traveled in 5 seconds.

Time (seconds)	0	1	2	3	4	5
Velocity (ft/sec)	3	5	12	22	25	28

Table 1: Velocity in ft/sec in 1 second intervals

- (a) What is the minimum distance traveled in each of the following time periods?

$t = 0$  to  $t = 1$

$t = 1$  to  $t = 2$

$t = 2$  to  $t = 3$

$t = 3$  to  $t = 4$

$t = 4$  to  $t = 5$

Add to find an estimate of the minimum distance traveled in 5 seconds.

(b) What is the maximum distance traveled in each of the following time periods?

t = 0 to t = 1

t = 1 to t = 2

t = 2 to t = 3

t = 3 to t = 4

t = 4 to t = 5

Add to find an estimate of the maximum distance traveled in 5 seconds.

(c) What is the difference in your two estimates?

5. Suppose additional velocity readings are obtained and compiled with the above information to give Table 2.

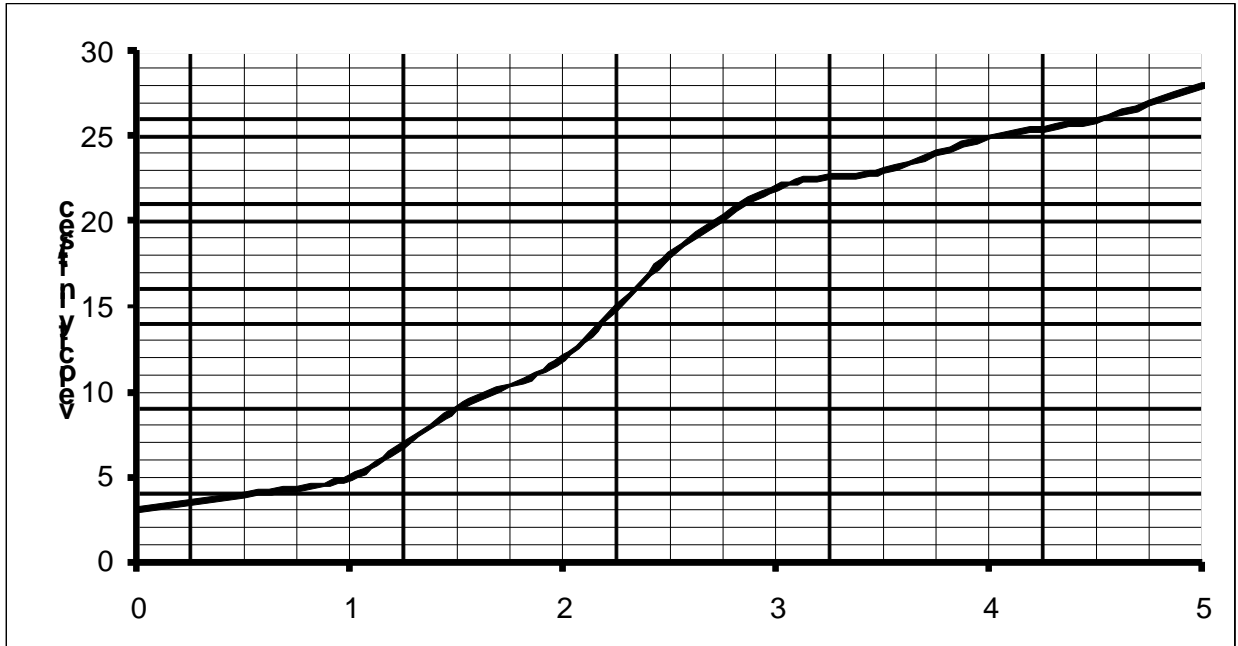
time	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5
vel.	3	4	5	9	12	18	22	23	25	26	28

Table 2: Velocity in ft/sec in one-half second intervals

(a) Recalculate the lower and upper estimates for the distance traveled in light of this new information. Calculate the new difference in your estimates.

OVER →

- (b). The data points in Table 2 are graphed below and connected to make a smooth increasing curve. How can your estimate of the minimum distance traveled be represented on the graph? What about the maximum distance traveled? How can the difference between these estimates be represented on the graph?



- (c) If velocity readings were obtained for every 0.1 second, by how much would your lower and upper estimates differ? What if readings were obtained for every .01 second?
- (d) Explain why you can calculate the distance traveled to any desired degree of accuracy if you have access to velocity readings at every instant during the 5 second interval.