Math 160 – Section 1.5 – Differentiation Techniques

THE POWER RULE

For any real number r, $f(x) = x^r$ **then** $f'(x) = r \cdot x^r$

With another notation: $\dfrac{d(x^r)}{dx} = r \bullet x^{r-1}$

Finding a derivative reduces the exponent of x by 1.

Examples:

Given $f(x)$,	Find $f'(x)$ Write answers with positive exponents
1. $f(x) = x^6$	
2. $f(x) = x^3$	
3. $f(x) = x^{-5}$	
4. $f(x) = x^{-1}$	
5. $f(x) = x^{-1/2}$	
6. $f(x) = x^{4/5}$	
7. $f(x) = x^{8/3}$	
8. $f(x) = x^{-3/4}$	

In order to use the power rule, sometimes we need to rewrite the functions using the laws of exponents. Here are some equivalent expressions:

$\sqrt[n]{x^m} = x^{\frac{m}{n}}$	$\frac{c}{\sqrt[n]{x^m}} = cx^{-\frac{m}{n}}$	$\frac{c}{x^n} = cx^{-n}$

Given f(x)	Rewrite f(x) using laws of exponents	Find $f'(x)$ Write answers with positive exponents
1. $f(x) = \sqrt[5]{x^3}$		
2. $f(x) = \sqrt{x}$		
3. f(x)= $\frac{1}{x^8}$		
4. $f(x) = \frac{1}{\sqrt{x}}$		

THE CONSTANT RULE

For any constant, c, $\frac{d}{dx}(c) = 0$**.**
That is, the derivative of a constant function is zero.

Examples: Find the derivatives of the following functions. Interpret the meaning.

1. y = 5,

2. f(x) = 8,

Math 160 – Section 1.5 – More Rules for Differentiation

<u>THE CONSTANT MULTIPLE RULE</u>

If k is a constant and f is differentiable, then so is the function kf(x) and

$$\frac{d}{dx}[k\bullet f(x)] = k\bullet\frac{d}{dx}[f(x)]$$

That is, the derivative of a constant times a function is the constant times the derivative of the function.

Examples:

Given $f(x)$,	Find $f'(x)$ Write answers with positive exponents
1. $f(x) = 3x^5$	
2. $f(x) = \frac{1}{2}x^{-6}$	
3. $f(x) = \frac{3}{x^2}$ rewrite as a power of x on the numerator then, find the derivative	
4. $f(x) = 7\bullet\sqrt[4]{x^5}$ rewrite as a power of x then, find the derivative	

THE SUM - DIFFERENCE RULE

If f(x) and g(x) are differentiable, so are the sum and difference functions S(x) = f(x) + g(x), and D(x) = f(x) – g(x)

$S'(x) = f'(x) + g'(x)$ or, written another way,

$$\frac{d}{dx}[f(x)+g(x)] = \frac{df}{dx} + \frac{dg}{dx}$$

So, the derivative of a sum is the sum of the separate derivatives.
The derivative of a difference is the difference of the derivatives.

Examples:

Given $f(x)$, Rewrite f if necessary	Find $\frac{dy}{dx}$; Find $f'(x)$; Find $\frac{d}{dx}(f(x))$
1. $f(x) = 2x^2 + 7x - 3$	$\frac{d}{dx}(f(x)) =$
2. $y = 3x^{-1} + 4x^{1/2} - 8x + 3$	$\frac{dy}{dx} =$
3. $f(x) = \frac{4}{x} + 3\sqrt{x^3} - 5x + 1$	$f'(x) =$
4. $y = \frac{-2}{\sqrt{x}} + \sqrt[3]{x^2} =$	y ‘ =
5. $f(x) = \frac{6x^5 - 8x^3}{2x^2}$ rewrite as a polynomial by dividing, then, find the derivative	

EVALUATING DERIVATIVES – FIN DING THE SLOPE OF THE LINE TANGENT TO THE GRAPH AT A GIVEN X

Examples:

1. Given $f(x) = 3x^2 - 5x + 2$, find $f'(2)$	Interpretation
2. Given $f(x) = \frac{3}{x} + 2\sqrt{x}$, find $f'(1)$	Interpretation
3, Given $y = \frac{5}{x^2}$, find $\left.\frac{dy}{dx}\right\vert_{x=-1}$	Interpretation
3, Find $\left.\frac{d(x^2+x)}{dx}\right\vert_{x=0}$	

Math 160 – Sections 1.5 Applications –
Similar applications should be solved for sections 1.6 and 1.7

1) a) Find the derivative of $f(x) = 3x^2 - 4x$ at the point P(1, -1).

b) What is the geometric meaning of the number found in part a)?

2) Find the slope of the line tangent to the graph of $f(x) = 4x^2 + 5x - 3$ when x = 3

3) Find the equation of the line tangent to the graph of $f(x) = \frac{1}{x}$ at the point where x = 1.

4) Given the function $f(x) = x^4 - 8x^2 - 9$, find the values of x where the derivative is zero.

5) Given the function $f(x) = x^3 - 3x$, find the **points** where the graph has a horizontal tangent line.

6) Use calculus to find the lowest **point** on the graph of the function $f(x) = 3x^2 + 4x - 3$.

7) Derivative as a rate of change - Given the function $f(x) = 4x^2 + 3x - 1$

a. Write the function that gives the rate at which f is changing with respect to x at any point.

b. At what rate is f changing with respect to x when x = -1?

c. Is the function increasing or decreasing as it passes through x = -1? How do you know?

d. At what rate is f changing with respect to x when x = 1? Is the function increasing or decreasing? How do you know?

e. What is the actual change of the function as x changes from 1 to 2?