

**Math 160 – Section R.4 – Slope of a Line, Average Rate of Change, Difference quotient**

In each of the problems given below, find the slope of the line going through the points P and Q. Some answers will be numbers, others will be expressions. Simplify the final answer as far as

possible.  $m = \frac{y_2 - y_1}{x_2 - x_1}$

**Example 1:**

Find the slope of the line going through

$P(3,14)$

$Q(5,17)$

$$m = \frac{17 - 14}{5 - 3} = \frac{3}{2}$$

**Example 2:**

Find the slope of the line going through

$P(3,14)$

$Q(c, f(c))$

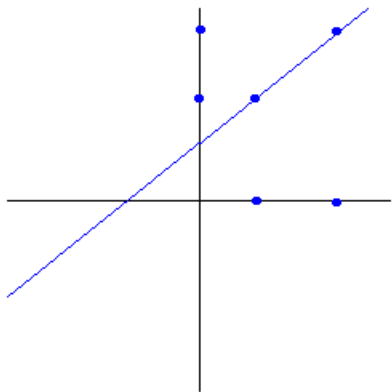
$$m = \frac{f(c) - 14}{c - 3}$$

Since we don't know the function, we can't simplify this one.

**Now you try it:**

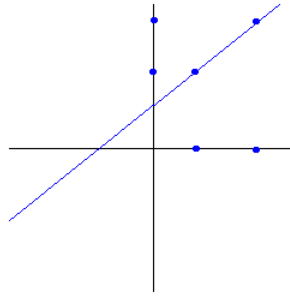
- 1)  $P(2,5)$   
 $Q(3,10)$

Also, label all 6 dots in the given graph.



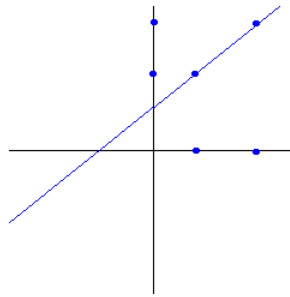
2)  $P(2,5)$   
 $Q(x, f(x))$

Also label all six dots in the given graph.



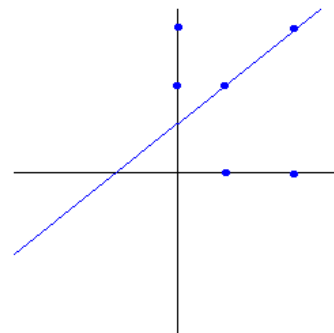
3)  $P(2, f(2))$   
 $Q(4, f(4))$

Also label all six dots in the given graph.



4)  $P(x, f(x))$   
 $Q(x+h, f(x+h))$

Also, label all six dots in the given graph.



**Example 3:**

For the given function, find each of the following:

$$f(x) = 2x^2 + 3x - 2$$

a) Find

$$\begin{aligned} f(x+h) &= 2(x+h)^2 + 3(x+h) - 2 = \\ &= 2x^2 + 4xh + 2h^2 + 3x + 3h - 2 \end{aligned}$$

b) Find

$$\begin{aligned} f(x+h) - f(x) &= (2x^2 + 4xh + 2h^2 + 3x + 3h - 2) - (2x^2 + 3x - 2) = \\ &= 2x^2 + 4xh + 2h^2 + 3x + 3h - 2 - 2x^2 - 3x + 2 = \\ &= 4xh + 2h^2 + 3h \end{aligned}$$

c) Find

$$\frac{f(x+h) - f(x)}{h} = \frac{4xh + 2h^2 + 3h}{h} = 4x + 2h + 3$$

**Note 1:** The answer to part c, is an expression for the slope of the line containing the points

$$P(x, f(x))$$

$$Q(x+h, f(x+h))$$

**Note 2:** The expression  $\frac{f(x+h) - f(x)}{h}$  is called the difference quotient.

Geometrically, the difference quotient represents the slope of the secant line connecting the points P and Q labeled in Note 1.

**Now you try it:**

**5) Given  $f(x)$ , find each of the following:**

$$f(x) = 3x^2 + 4x + 1$$

a) Find

$$f(x+h) =$$

b) Find

$$f(x+h) - f(x) =$$

c) Find

$$\frac{f(x+h) - f(x)}{h} =$$

**6) Given  $f(x)$ , find each of the following:**

$$f(x) = 2x^2 - 5x - 8$$

a) Find

$$f(x+h) =$$

b) Find

$$f(x+h) - f(x) =$$

c) Find

$$\frac{f(x+h) - f(x)}{h} =$$