

Some practice for exam 2

Also study handouts, quizzes, and homework problems

Read the study guide for exam 2 and make sure you know how to handle all of the topics

Solve the problem.

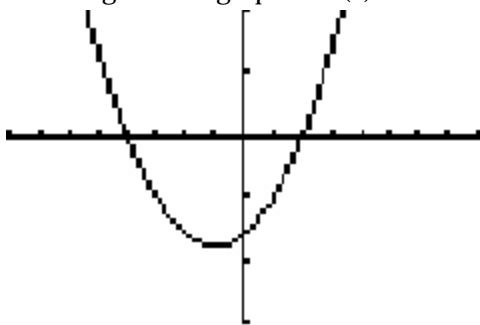
- 1) The annual revenue and cost functions for a manufacturer of precision gauges are approximately $R(x) = 500x - 0.01x^2$ and $C(x) = 120x + 100,000$, where x denotes the number of gauges made.
 - a) Write the marginal revenue function.
 - b) How many gauges should be produced and sold in order to produce the maximum profit?
 - c) What is the maximum annual profit?

- 2) A company is constructing an open-top, square-based, rectangular metal tank that will have a volume of 25 ft^3 .
 - a) What dimensions yield the minimum surface area? Round to the nearest tenth, if necessary.
 - b) What is the minimum surface area? Sketch a graph of the Surface area function - no calculator needed.
 - c) Suppose the cost of the bottom is \$5 per square foot and each of the sides has a cost of \$4 per square foot, what are the dimensions of the cheapest box?
 - d) What is the cost of this box? Sketch a graph of the cost function - no calculator needed.

- 3) If the price charged for a bolt is p cents, then x thousand bolts will be sold in a certain hardware store, where $p = 132 - \frac{x}{20}$. How many bolts must be sold to maximize revenue? What is the optimal price per bolt?
What is the maximum revenue?

- 4) The gross annual earnings of a certain company are $f(t) = \sqrt{10t^2 + t + 236}$ thousand dollars t years from now.
 - a) At what rate will the gross annual earnings of the company be growing 3 years from now? Your answer must contain units.
 - b) What is the meaning of your answer to part (a) within the context of the problem? Your explanation must contain the words "increasing, or decreasing" and proper units.

- 5) You are given the graph of $f'(x)$. The x -scale used is 1.



Complete the following:

- a) The function has a HTL at $x = \dots\dots\dots$
- b) The function is increasing on the interval(s) $\dots\dots\dots$
- c) The function is decreasing on the interval(s) $\dots\dots\dots$
- d) The second derivative is zero when $x = \dots\dots\dots$,
which means we possibly have an inflection point at $x = \dots\dots$
- e) The function is concave up on the interval (s) $\dots\dots\dots$
- f) The function is concave down on the interval (s) $\dots\dots\dots$

- 6) Sketch the graph of a function with domain all real numbers such that $f'(x) > 0$ for all x and $f''(x) < 0$ for all x .
- 7) Given that $f(3) = 7$, $f'(3) = 0$ and $f''(3) = -2$
What does this information tell you about the function? Sketch the graph of the function around the given point.
- 8) The profit (in dollars) generated by the sale of x units of a certain product is given by $P(x) = -0.5x^2 + 22x - 98$, find each of the following: (numbers should have appropriate units)
- a) $P(15) =$
- b) $P(16) =$
- c) $P(16) - P(15) =$
- d) $\frac{P(16) - P(15)}{16 - 15} =$
- e) $P'(15) =$
- f) $P(15) + P'(15) =$

Use the answers from parts a-f (ABOVE) to complete the following statements. (Use appropriate units)

- (1) When 15 units are sold, the profit is increasing/decreasing (circle one) at a rate of _____
- (2) The ACTUAL profit generated by the sale of 16 units is _____
- (3) An ESTIMATE for the profit generated by the sale of 16 units is _____
- (4) The average rate of change of profit over the interval $[15, 16]$ is _____
- (5) An ESTIMATE for the change in profit over the interval $[15, 16]$ is _____
- (6) The marginal profit when we sell 15 units is _____
- (7) The ACTUAL change in profit over the interval $[15, 16]$ is _____
- (8) What is an estimate for the profit produced by the sale of the 16th unit? _____
- (9) Estimate $P(15.2)$

Solve the problem.

9) Assume that the temperature of a person during an illness is given by

$$T(t) = -0.1t^2 + 1.3t + 98.6, \quad 0 \leq t \leq 13,$$

where T = the temperature ($^{\circ}\text{F}$) at time t , in days after the onset of the illness.

a) What is the average rate of change of the temperature of the patient during the first 3 days? Interpret in words within context. Use the words increasing/decreasing in your interpretation and proper units.

b) What is the instantaneous rate of change of the temperature of the patient six days after the onset of the illness? Interpret in words within context. Use the words increasing/decreasing in your interpretation and proper units. What kind of indication is the sign of the derivative providing to the doctor?

10) The first and second derivatives of a function $f(x)$ are given below. Notice: I am not providing $f(x)$ so you can't find maximum and minimum with the calculator. Because of this, you will not be able to find points on the graph.

$$f'(x) = 6x^2 + 6x - 12 \quad \text{and} \quad f''(x) = 12x + 6$$

a) Find first order critical numbers.

b) Use the first derivative test to determine whether $f(x)$ has a maximum, a minimum or neither at each first order critical number. (Must show a sign chart)

c) Use the above information to answer the following:

$f(x)$ has / does not have a relative maximum at $x = \underline{\hspace{2cm}}$ because

$f(x)$ has / does not have a relative minimum at $x = \underline{\hspace{2cm}}$ because

Give the intervals of increase of $f(x)$

Give the intervals of decrease of $f(x)$

d) Find the second order critical numbers and study concavity of the function. (Must show a sign chart)

e) Use the information from part (d) to answer the following

$f(x)$ has / does not have an inflection point at $x = \underline{\hspace{2cm}}$ because

Give intervals of concavity up for $f(x)$

Give intervals of concavity down for $f(x)$

f) Sketch a possible graph the function

Answer Key

Testname: REV-EXAM2

- 1) a) $R' =$
b) $P = R - C$, then find P' , then solve $P' = 0$ and get 19,000 gauges, c) \$3,510,000
- 2) a) 3.7 ft by 3.7 ft. by 1.8 ft; b) 40.7 sq.ft; c) 3.4 ft by 3.4 ft by 2.2 ft; d) \$175.44
- 3) 1320 thousand bolts; 66 cents; \$871.20
- 4) -4 thousand dollars per year (decreasing)
- 5) a) $x = -4$, $x = 2$; b) $(-8, -4) \cup (2, 8)$; c) $(-4, 2)$; d) $x = -1$; e) $(-1, 8)$; f) $(-8, -1)$
- 6)
- 7)
- 8)
- 9)
- 10)