

MA 110 SECTION 5.1: INEQUALITIES IN TWO VARIABLES

**HOMEWORK: 1, 7, 13, 21, 25, 29, 33, 39, 41**

1. We are building the skills to solve applications using linear programming. In linear programming the solution of a problem is found by examining the graph of a system of linear inequalities.

2. **Linear inequalities have the form:**

$$\begin{array}{l} Ax + By \leq C \quad \text{OR} \quad y \leq mx + b \\ Ax + By \geq C \quad \text{OR} \quad y \geq mx + b \end{array}$$

3. In the linear programming problems we will complete, the linear inequalities are never strict (they always include the "or equal to" portion of the inequality) Hence, we can always graph the line first and then shade the inequality. You always need to plot two points to graph a line. In linear programming problems (that we will soon complete) the x-intercepts and y-intercepts of inequalities are often part of the solution. Thus, in practice for linear programming, we will graph lines by finding the x-intercept and y-intercept.

4. EXAMPLE:  $y \geq x + 1$

To find the intercepts, temporarily disregard the inequality and consider the equation  $y = x + 1$ .

How do you find the x-intercept of  $y = x + 1$ ?

$$\text{Let } y = 0 \text{ and solve } 0 = x + 1 \rightarrow x = -1 \rightarrow (-1, 0)$$

How do you find the y-intercept of  $y = x + 1$ ?

$$\text{Let } x = 0 \text{ and solve } y = 0 + 1 \rightarrow y = 1 \rightarrow (0, 1)$$

Plot these two points and connect with a line.

A line divides a coordinate plane into two half-planes. One half plane represents the points that satisfy  $y > x + 1$  (the other half-plane represents the points that satisfy  $y < x + 1$ ). How we determine which half-plane is which? When the inequality is in the form  $y > mx + b$ , intuitively, the half-plane above the line corresponds to  $>$  and the half-plane below the line corresponds to  $<$ . Let's draw some light arrows above our line. Note, in future linear programming problems you will draw light arrows before you complete a heavy shading – so this is a good habit to practice.

If you find it hard to determine which region is above or below the line or are uncomfortable with this intuitive procedure, we can also find the appropriate half-plane by using a test point. Let's choose a point that we are sure is above the dotted line  $y = x + 1$ .  $(0, 2)$  is fine. Substitute 0 for x and 2 for y in the inequality  $y > x + 1$   
 $2 > 0 + 1 \rightarrow 2 > 1 \rightarrow \text{true}$

So we can shade the corresponding half-plane.

5. EXAMPLE:  $4x - 8y \geq 32$

Temporarily consider  $4x - 8y = 32$ .

Find the x-intercept  $\rightarrow 4x = 32 \rightarrow x = 8 \rightarrow (8, 0)$

Find the y-intercept  $\rightarrow -8y = 32 \rightarrow y = -4 \rightarrow (0, -4)$

Plot these two ordered pairs and connect with a line. Now which half-plane should be shaded? Note, that the inequality is not in the form  $y \leq mx + b$ , so to use the intuitive process, you must convert it first. Since, our graphing calculators can only graph lines in the form  $y = mx + b$ , this is a good habit to practice.

$$4x - 8y \geq 32 \rightarrow -8y \geq -4x + 32 \rightarrow y \leq (1/2)x - 4$$

Thus, now we see that we should shade the half-plane BELOW the line. You can also check this with a test point that is definitely above the line,  $(0, -5)$  is good.

$$4x - 8y \geq 32$$

$$4(0) - 8(-5) \geq 32$$

$$40 \geq 32 \text{ true}$$

$$y \leq (1/2)x - 4$$

$$-5 \leq (1/2)(0) - 4$$

$$-5 \leq -4 \text{ true}$$

Thus, we should shade below the line.

6. **SPECIAL LINES AND INEQUALITIES:**

A. Horizontal lines (constant functions)  $f(x) = b$  or  $y = b$ .

How to graph  $y = b$ ,  $y \geq b$ ,  $y \leq b$

Example:  $y \leq 6$  Temporarily consider  $y = 6$ . How do you graph this line? In this case, it is quite obvious which half-plane is above or below the line  $y = 6$ . Obviously, below the line values of  $y$  are less than 6. Thus, we shade below the line.

B. Vertical lines  $x = a$

Example:  $x \leq 5$  Temporarily consider  $x = 5$ . How do you graph this line? Now, there is no such thing as above or below this line. A test point will reveal the answer. Intuitively, points to the right of the line will have  $x$  values greater than 5 and points to the left of the line will have values less than the line.

7. Class: Graph  $3x + 4y \leq 12$

8. Writing linear inequalities
- A. Labor costs for a farmer are \$55 per acre for corn and \$45 per acre for soybeans. How many acres of each crop should the farmer plant if she wants to spend no more than \$6,900 on labor?
- B. A farmer wants to use two brands of fertilizer for her soybean crop. Brand A contains 18% nitrogen, 24% phosphate, and 12% potash. Brand B contains 5% nitrogen, 10% phosphate, and 15% potash.
- i. How many pounds of each brand of fertilizer should she add to each acre if she wants to add at least 50 pounds of phosphate to each acre?
  - ii. How many pounds of each brand of fertilizer should she add to each acre if she wants to add at most 60 pounds of potash to each acre?