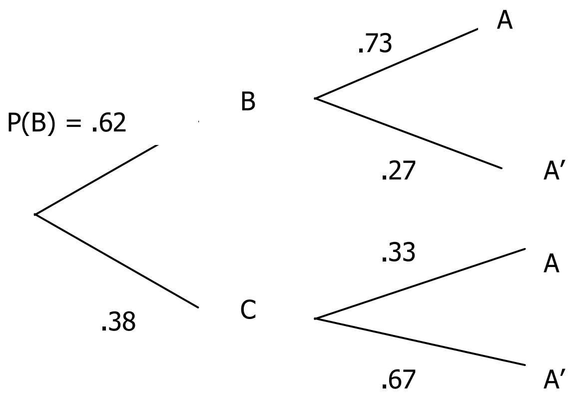
MA 110 SECTION 8.4: PROBABILITY TREES & BAYES THEOREM

HOMEWORK: 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 47, 49, 51

1. Recall: $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$ so $\dfrac{P(A \cap B)}{P(B)} = P(A|B)$

   Multiplying both sides by P(B) we get the **PRODUCT RULE:** $P(A \cap B) = P(B)\,P(A|B)$

2. Given a probability Tree



Computations using the probability tree

A. $P(B \cap A) = P(B)P(A|B) =$

B. $P(B \cap A') = P(B)P(A'|B) =$

C. $P(C \cap A) = P(C)P(A|C) =$

D. $P(C \cap A') = P(C)P(A'|C) =$

E. $P(B) =$

F. $P(C) =$

G. $P(A) = P(B \cap A) + P(C \cap A) =$

H. $P(A') = P(B \cap A') + P(C \cap A') =$

In general, P(A) = sum of all branch probabilities leading to A

I. $P(B|A) = \dfrac{P(B \cap A)}{P(A)} =$

J. $P(C|A') = \dfrac{P(C \cap A')}{P(A')} =$

In general $P(B|A) = \dfrac{\text{Product of branch probabilities leading to A from B}}{\text{Sum of all branch probabilities leading to A}}$

2. **EXAMPLES:**

A. In a given country, records show that of the registered voters 45% are Democrats, 35% are Republicans, and 20% are independents. In an election 70% of the Democrats, 40% of the Republicans, and 80% of the independents voted in favor of a parks and recreation bond proposal.

i. Draw a probability tree that displays this information.

If a registered voter chosen at random is found to have voted in favor of the bond, what is the probability that the voter is

ii. a Republican?

iii. An independent?

iv. A Democrat?

B. A test for tuberculosis was given to 1,000 subjects, 8% of whom were known to have tuberculosis. For the subjects who had tuberculosis, the test indicated tuberculosis in 90% of the subjects, was inconclusive for 7%, and indicated no tuberculosis in 3%. For the subjects who did not have tuberculosis, the test indicated tuberculosis in 5% of the subjects, was inconclusive for 10% and indicated no tuberculosis in the remaining 85%.

i. Draw a probability tree that displays this information.

ii. What is the probability of a randomly selected person having tuberculosis given that the test indicates tuberculosis?

iii. Of not having tuberculosis given that the test was inconclusive?

iv. Of testing positive given the person has tuberculosis?

v. Of not having tuberculosis given the test was positive (False Positive)?

vi. Of having tuberculosis given the test was negative (False negative)?