MA 130 SECTION 4.1 FACTORS **HOMEWORK: 1 – 25 odd**

OBJECTIVES:

- ❖ Prove or disprove conjectures about factors

1. If A and B are whole numbers, with A $\neq$ 0, then A is a **factor** or **divisor** of B, written A|B, if and only if there is whole number C such that A $\bullet$ C = B.

   This statement means that A divides B evenly. It also means C is the quotient of B $\div$ A.

   Using the definition:

   A. Does 2|12?

   B. Does 3|12?

   C. Does 5|12?

   D. Does 12|4?

2. **The Divisibility-of-a-Sum Theorem** For any whole numbers A, B, C, with A $\neq$ 0, if A|B and A|C, then A| (B + C).

   If 5|20 and 5|40 then 5|60.

   If 3|15 and 3|60 then _____.

3. **The Divisibility-of-a-Difference Theorem** For any whole number A, B, and C, with A $\neq$ 0, and C > B, if A|B and A|C, then A| (C - B).

   If 5|60 and 5|40, then 5|20.

   If 7|49 and 7|91, then _____.

4. **The Divisibility-of-a-Product Theorem** For any whole numbers A, B, C, with A $\neq$ 0, if A|B then A| BC.

   Is the following an instance of the Divisibility-of-a-Product Theorem?

   If 6|42 then 6|420.

5. A **perfect number** equals the sum of all its factors that are less than itself. For example, 6 is a perfect number since $6 = 1 + 2 + 3$.

6. A **deficient number** occurs when the sum of all the factors that are less than itself are less than the number. For example, 9 is a deficient number since $1 + 3 = 4 < 9$.

7. An **abundant number** occurs when the sum of the factors that are less than itself are more then the number. For example, 12 is an abundant number since $1 + 2 + 3 + 4 + 6 = 16 > 12$.

8. Problems to try: Decide whether each statement is true of false. If it is true, prove it. If it is false, give a counterexample. Assume that A, B, C, and D are whole numbers, with $A \neq 0$.

   A. If A|B, then A|(B + 1)

   B. 1|B for all B.

   C. The product of two odd whole numbers is odd. Hint: If C is odd, then $C = 2w + 1$ for some whole number w.

   D. Test each of the following numbers to see if it is perfect, deficient, or abundant.

   (a) 120 (b) 315 (c) 496