MA 181 SECTION 2.3 Calculating limits using the limit laws

1. **Familiarize yourself with THE LIMIT LAWS on pages 104 – 110.**

The function given is a rational function defined at 2, so we could just "plug" 2 for x. Note Direct Substitution Property p. 117. To utilize a limit law note:

$$\lim_{x\to 2} \frac{2x^2 + 1}{x^2 + 6x - 4} = \frac{\lim_{x\to 2} 2x^2 + 1}{\lim_{x\to 2} x^2 + 6x - 4} =$$

$$= \frac{2(2)^2 + 1}{(2)^2 + 6(2) - 4} = \frac{2 \lim_{x\to 2} x^2 + \lim_{x\to 2} 1}{\lim_{x\to 2} x^2 + 6 \lim_{x\to 2} x - \lim_{x\to 2} 4} = \frac{9}{12} = \frac{3}{4}$$

2. **CAUTIONS ABOUT USING THE LIMIT LAWS**

Note: The hypothesis of the Limit Laws states that the limit of f(x) and g(x) exist at a. If the limit of f(x) and/or g(x) dne then the limit laws do not apply and the limits of f(x) + g(x) and/or f(x) * g(x) may or may not exist.

3. **EXAMPLE:**

$$f(x) = \begin{cases} 0 & x < 0 \\ 1 & x > 0 \end{cases} \qquad g(x) = \begin{cases} 1 & x < 0 \\ 0 & x > 0 \end{cases}$$

Examine the limit as x ➔ 0 of A. (f+g)(x)

B. (f*g)(x)

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5. **MORE EXAMPLES:**

A. $\lim_{x \to 0} \|x\| \sin x = 0$, but you can't use the Product Law, since

$\lim_{x \to 0} \|x\|$ is undefined.

B. The limit laws will not directly apply. However, with a little algebra we can rewrite the function and apply the limit laws to confirm the following limit.

$$\lim_{t \to 0} \frac{\sqrt{t^2 + 9} - 3}{t^2} = \frac{1}{6}$$

6. CLASS WORK: Show that $\lim_{x \to 1} \frac{\sqrt{x + 3} - 2}{x - 1} = \frac{1}{4}$

7. What can you do if you can't use the limit laws and you can't use a simple algebraic procedure to help find the limit?

THE SQUEEZE THEOREM is sometimes applicable.

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