MA 181 SECTION 2.4 CONTINUITY

1. DEFINITION: A function f is continuous at a number a if

$\lim_{x \to a} f(x) = f(a)$, which implies that

i. f(a) is defined
ii. $\lim_{x \to a} f(x)$ exists
iii. $\lim_{x \to a} f(x) = f(a)$

The function is not continuous at a number, a, if any of the three above statements are not true.

2. FUNCTIONS KNOWN TO BE CONTINUOUS:

A. Polynomials
B. Rational functions (over their domain)
C. Trigonometric functions (over their domain)
D. Exponential functions
E. Logarithmic functions (over their domain)

3. SOME EXAMPLES OF DISCONTINUITY:

A. Removable Discontinuity

$h(x) = \frac{x^2 + x - 12}{x - 3}$ has a removable discontinuity at x = 3

B. Jump Discontinuity

$g(x) = \begin{cases} x - 1 & \text{if } x \le 1 \\ x + 2 & \text{if } x > 1 \end{cases}$ has a jump discontinuity at x = 1

C. Infinite discontinuity

f(x) = tan x is not continuous at $x = \pi/2$

4. DEFINITION (2): PAGE 115 Continuous from the left/right.

E.g. $y = \sqrt{x}$

5. DEFINITION (3): PAGE 115 Continuous on an interval. See Example 4

6. THEROR€MS 4 – 9 ON pages 116 - 119 are very helpful for us. One way knowledge about continuity is helpful is that if we know a function is continuous at a number a, then we can evaluate the limit of that function at a by finding f(a).

7. Examples: page 122: #28. Thms. 8 & 4

$$\lim_{x \to \pi} \sin(x + \sin x) = \sin\left(\lim_{x \to \pi} (x + \sin x)\right) = \sin(\pi + \sin \pi) = \sin(\pi + 0) = 0$$

8. Another helpful property of continuous functions is given in the IVT (p. 120: Thm. 10)

   A. Graphing calculator assumption: E.g. y = tan x

   B. Is there a number that is exactly 1 more than its cube?

      $Y = x - x^3 - 1$ has one real solution.

9. CLASS:

Find all points for which f(x) is not continuous.

$$f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ \sin x & \text{if } 0 \le x < \pi/2 \\ x & \text{if } x \ge \pi/2 \end{cases}$$