MA 181 SECTION 2.5 LIMITS INVOLVING INFINITY

1. INFINITE LIMITS

DEFINITION: The notation $\lim_{x \to a} f(x) = \infty$

Means that the values of f(x) can be made arbitrarily large by taking x sufficiently close to a.

E.g. $\lim_{x \to 0} \frac{1}{x^2} = \infty$ GRAPH/LOCAL PROGRAM

What does it mean that values of $f(x) = 1/x^2$ can be made arbitrarily large?

Choose a large value (say 1,000,000) how close to zero must x be so that f(x) is as large as 1,000,000 (or larger)?

2. NOTE:

i. This does not mean infinity is a number.
ii. This still implies that the limit does not exist – it is just a particular way in which the limit does not exist.
iii. There are several ways of reading the notation for such a limit. See page 124.

3. **ASYMPTOTES (& LIMITS INVOLVING INFINITY)**

A. Vertical Asymptote (definition) page 125

Example: $f(x) = \frac{x + 5}{x^2 - 25}$

4. AN EXAMPLE OF A NON-RATIONAL FUNCTION WITH VERTICAL ASYMPOTOTES:

Y = cot x has infinitely many vertical asymptotes.

x = … -3π, -2π, $-\pi$, 0, π, 2π, 3π, … x = nπ

5. Find the following limits:

A. $\lim_{x \to 3} \frac{1}{x - 3}$

B. $\lim_{x \to 3} \frac{1}{(x - 3)^2}$

C. $\lim_{x \to 3} \frac{-4}{(x - 3)^2}$

6. **LIMITS AT INFINITY**

DEFINITION: Let f be defined on some interval (a, ∞). Then $\lim_{x \to \infty} f(x) = L$ means that the values of f(x) can be made as close to L as we like by taking x sufficiently large.

The answer to limits at infinity is often reference as the END behavior of a function.

7. Example f(x) = $\frac{x + 5}{x^2 - 25}$

8. RATIONAL FUNCTIONS & ASYMPTOTES

Find & generalize the limits at infinity for the following functions: (discuss degrees)

A. $g(x) = \dfrac{3x + 5}{x - 4}$

B. $h(x) = \dfrac{3x^2 + 5}{x - 4}$

C. $j(x) = \dfrac{3x^3 + 5}{x - 4}$

D. $f(x) = \dfrac{x + 5}{x^2 - 25}$

9. INFINITE LIMITS AT INFINITY: Note example 9 page 132

10. MORE EXAMPLES:

A. $\lim_{x \to \infty} \frac{x + 2}{\sqrt{9x^2 + 1}}$

B. $\lim_{x \to \infty} \frac{\sin^2 x}{x^2}$

C. $\lim_{x \to \infty} \tan^{-1}\left(x^4 - x^2\right)$

D. $\lim_{x \to \infty} e^{-x^2}$

E. $\lim_{x \to \infty} \left(\sqrt{16x^2 - 5x} - \sqrt{16x^2}\right)$