MA 181 SECTION 2.6 DERIVATIVES & RATES OF CHANGE

1. The tangent line to the curve y = f(x) at the point P(a, f(a)) is the line through P with slope $m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

provided the limit exits.

Use this definition to find the slope of the tangent line to the curve $y = x^2 + 4$ at the point (1, 5). Then write the equation of the tangent line.

2. The slope of the tangent line to a curve at a point is referred to as the slope of the curve at that point. The idea is that every function is locally linear. That is, if we zoom in close enough the curve is indistinguishable from the tangent line. Look at the curve $y = x^2 + 4$ and the tangent line to the curve at (1, 5) in the following windows.

1. [0, 5] x [0, 10]
2. [0, 3] x [3, 6]
3. [.65, 1.4] x [4.67, 5.4]
4. [0.9, 1.1] x [4.9, 5.1]

3. There is another expression for the slope of a tangent line. Consider the two points, (a, f(a)) and (a + h, f(a + h)). The slope of the line through these two points is given by

$\frac{f(a + h) - f(a)}{(a + h) - a} = \frac{f(a + h) - f(a)}{h}$. This is the slope of the secant line

to f(x) through the points (a, f(a)) and (a + h, f(a + h)). We saw in section 2.1, that the slope of the tangent line is the limit of the slopes of the secant lines. So as h gets close to zero, this expression gets close to the slope of the tangent line.

Thus, the slope of the tangent line at a point (a, f(a)) can be given by

$$m = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

4. Find the slope of the tangent line to $y = x^2 + 4$ at (1, 5) using the 2^{nd} definition.

5. **VELOCITIES** In section 2.1, we saw that the average velocity is the displacement/time. See diagrams, p. 137. We can define the instantaneous velocity as the limit of the average velocities. Definition #3.

Example: The displacement (in meters) of a particle moving in a straight line is given by $s = t^2 - 8t + 18$.

A. The average velocity between times t and t + h is

$$\frac{s(t+h) - s(t)}{(t+h) - t} = \frac{(t+h)^2 - 8(t+h) + 18 - (t^2 - 8t + 18)}{h}$$

$$= \frac{t^2 + 2ht + h^2 - 8t - 8h + 18 - t^2 + 8t - 18}{h} = \frac{2ht + h^2 - 8h}{h}$$

$= (2t + h - 8)$ m/s

Use this formula to find the average velocity over the following time intervals.

i. [3, 4]

ii. [3.5, 4]

iii. [4, 5]

iv. [4, 4.5]

B. Find the instantaneous velocity when t = 4.

The instantaneous velocity at any time t is

C. Graph $s(t) = t^2 - 8t + 18$ on [0, 5] by [0, 3]

Use Draw Tangent to sketch the tangent line at t = 4.

6. The derivative of a function f at a number a, denoted by f′(a), is

$$f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

The tangent line to y = f(x) at (a, f(a)) is the line through (a, f(a)) whose slope is equal to f′(a), the derivative of f at a.

The derivative f′(a) is the instantaneous rate of change of y = f(x) with respect to x when x = a.

7. EXAMPLES:

If $g(x) = \sqrt{3x - 2}$, find g′(6) and use it to find an equation of the tangent line to the curve $y = \sqrt{3x - 2}$ at the point (6, 4).

APPLICATIONS

8. The number of bacteria after t hours in a controlled laboratory experiment is n = f(t).

A. What is the meaning of the derivative f′(5)? What are its units?

B. Suppose there is an unlimited amount of space and nutrients for the bacteria. Which do you think is larger, f′(5) or f′(10)? If the supply of nutrients is limited, would that affect your conclusion? Explain.

9. Life expectancy improved dramatically in the 20^{th} century. The table gives values of E(t), the life expectancy at birth (in years) of a male born in the year t in the United States. Interpret and estimate the values of E′(1910) and E′(1950).

t	E(t)	t	E(t)
1900	48.3	1960	66.6
1910	51.1	1970	67.1
1920	55.2	1980	70.0
1930	57.4	1990	71.8
1940	62.5	2000	74.1
1950	65.6		