MA 181 SECTION 2.7 THE DERIVATIVE AS A FUNCTION

1. The derivative of a function f, denoted by f′(x), is

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

2. Find the derivative of $f(x) = 5 - 4x + 3x^2$ using the definition.

3. A function f is differentiable at a if f′(a) exists. It is differentiable on an open interval (a, b) if it is differentiable at every number in the interval. (See definition 3 on page 150)

4. How can a function fail to be differentiable?

Case 1. Thm. 4 If f is differentiable at a, then f is continuous at a.

Thus, if a function is not continuous at a, then it can't be differentiable at a. (This is the contrapositive of Thm. 4)

See graph of g in problem #38 on page 157.

5. Case 2. We are not done with #38, but first we need to review other conditions in which a function is not differentiable.

Note: The converse of theorem 4 is not true. That is, f continuous at a does not imply f is differentiable at a.

y = |x| provides a counterexample to the converse of theorem 4, since f'(0) does not exist.

6. In fact, if the graph of f has a "corner", "kink" or "sharp point" then the graph of f has no tangent at that point and is not differentiable there.

7. Look again at page 157 #38

8. Case 3. A third possibility is that the curve has a vertical tangent line when x = a.

Example: $f(x) = x^{(1/3)}$

9. THE SECOND DERIVATIVE FUNCTION f" is the derivative of f '

See notation (and p. 153 - 154), second derivatives will be covered in section 2.8.