MA 181 SECTION 2.8: WHAT DOES f ' SAY ABOUT f?

The sign of a function's derivative is based on the behavior of the function's graph. The graph of the derivative of a function can also provide us with information about the original function.

1. **REVIEW:** It was observed that the slope of the tangent line to f at a point a had a positive slope if f was increasing on an interval that contained a. In fact, wherever f is increasing the slope of the tangent line is positive. Thus, if f is increasing on an interval, then $f'(x) > 0$ on that interval.

   NEW:

2. **REVIEW:** The slope of the tangent line to f at a point b had a negative slope if f was decreasing on an interval that contained b. Wherever f is decreasing the slope of the tangent line is negative. Thus, if f is decreasing on an interval, then $f'(x) < 0$ on that interval.

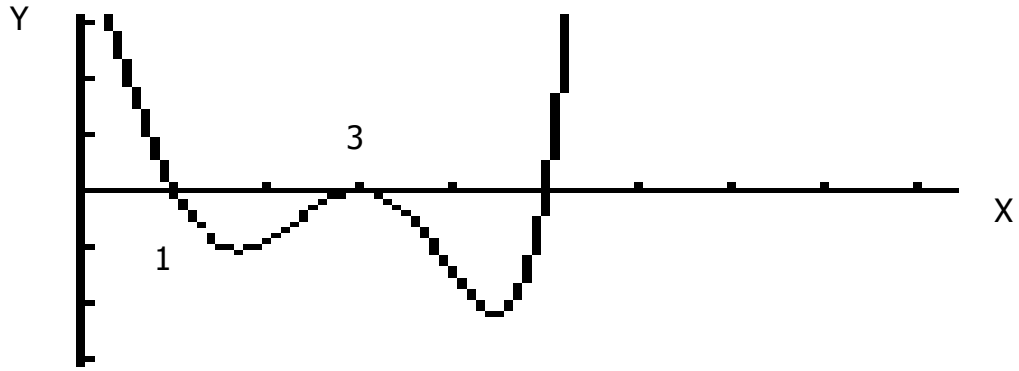
   NEW:

3. **FURTHERMORE:** If $f'(x)$ changes sign at $x = c$ (a value in the domain of f)

   AND $f'(c) = 0$ or $f'(c)$ is undefined,

   THEN f has a local extrema (local maximum or local minimum) at $x = c$.

4. The graph given below is $y = f'(x)$. Use it to answer the following questions. Assume f is defined on $(0, \infty)$.



   1. Where (on what intervals) is f decreasing?

   2. Where is f increasing?

   3. Where does f have a local minimum?

   4. Where does f have a local maximum?

   5. Sketch a possible graph of $y = f(x)$.

6. What does f" say about f? (Note the relationship between f" and f ' is the same as the relationship between f ' and f).

A. If f"(x) > 0 on an interval,

DRAW SAMPLES:

then
1. f ' is increasing on that interval
2. f is concave upward on that interval
3. the curve f(x) bends upward.

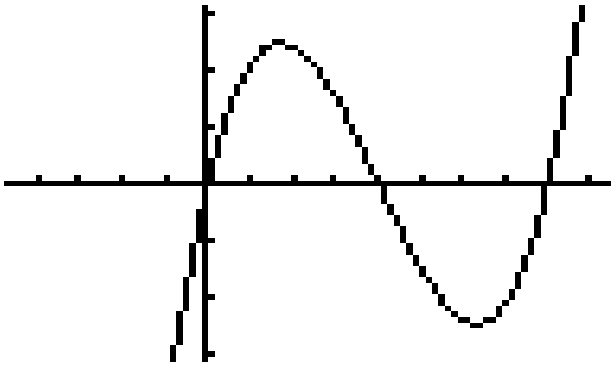
B. If f"(x) < 0 on an interval,

DRAW SAMPLES:

then
1. f ' is decreasing on that interval
2. f is concave downward on that interval
3. the curve f(x) bends downward.

** Note: Lines (linear functions) do not have concavity. **

C. An inflection point is a point where a curve changes its direction of concavity. An inflection point occurs at a = c when f" changes sign at x = c **AND** either f"(c) = 0 or f"(c) is undefined.



7. Use the graph of y = f"(x) above to answer the following questions. Assume f is defined for all real numbers.

1. Where is f(x) concave upward?

2. Where is f(x) concave downward?

3. Where does f(x) have inflection points?