Ma 181 SECTION 3.7: DERIVATIVES OF LOGARITHMIC FUNCTIONS

1. Consider the function $y = \log_a x$. So far, we have not found a formula for the derivative of this function. To find such a formula,

2. Recall that the inverse of a logarithmic function is an exponential function. Rewrite $y = \log_a x$ in exponential form as $x = a^y$ and use implicit differentiation to find $\frac{dy}{dx}$. Write your answer so that it is in terms of x only (not y). That is, for $x = a^y$, find $\frac{dy}{dx}$ in terms of x.

3. Now use this formula to find $\frac{d(\ln x)}{dx}$. (Recall that $\ln x$ means $\log_e x$.)

4. Use the formulas that you developed to differentiate each of the following functions.

A. $y = \frac{\ln x}{x^2}$ B. $y = \sin(\ln x)$ C. $y = \log_5 x$

5. Use the Chain Rule to extend the formulas to determine

A. $\frac{d[\ln(g(x))]}{dx}$ B. $\frac{d[\log_a (g(x))]}{dx}$

6. Use the formulas above to differentiate the following functions.

A. $y = \ln(x^3 + 2)$ B. $y = \ln(\sin x)$ C. $y = \log_3(5x + 2)$

7. There are three rules of logarithms which can be used to rewrite expressions involving logarithms so that these expressions do not contain products, quotients, or extended power functions. You should already be familiar with these rules. They are:

(1) $\log_a(MN) = \log_a M + \log_a N$

(2) $\log_a \frac{M}{N} = \log_a M - \log_a N$

(3) $\log_a M^r = r \log_a M$

Use one or more of these rules, as appropriate, to rewrite each of the following expressions **before differentiating** and then find the derivative.

A. $y = \ln(3x^4 + 5x)^7$ B. $y = \ln \frac{3x+1}{\sqrt{x^2+2}}$

8. How can you find the derivative of $y = x^x$? No, you can't use the power rule. Why?

There are two techniques that can be used to differentiate $y = x^x$.

TECHNIQUE #1: LOGARITHMIC DIFFERENTIATION

TECHNIQUE #2 $e^{\ln f(x)}$ "substitution"

TEXTBOOK REFERENCES:

A. page 224 Steps in Logarithmic Differentiation
B. page 224 Easier proof of Power Rule
C. page 225 The number e as a limit