MA 181 SECTION 3.9: LINEAR APPROXIMATIONS AND DIFFERENTIALS

1. The equation of the tangent line to the curve $y = f(x)$ at the point $(a, f(a))$ is

   $y = f(a) + f'(a)(x - a)$.

2. Note this equation is simply a version of the point-slope formula from algebra

   $y - y_0 = m(x - x_0)$ with the following "updates".

   1. The slope m is replaced with $f'(a)$
   2. The ordered pair (x_0, y_0) is replaced with $(a, f(a))$
   3. The equation is written in the $y =$ form.

3. **<u>LINEAR APPROXIMATIONS</u>:**

   The function $L(x) = f(a) + f'(a)(x - a)$ is called the linearization of f at a.
   $f(x)$ can be approximated by $L(x)$ for values of x close to a. **See fig. 1 p. 240**

4. The cost to produce x units of an item is given by

   $$C(x) = 920 + 2x - 0.02x^2 + 0.00007x^3$$

   Find a linear approximation for $C(x)$ at $x = 100$.

5. Approximate $C(110)$

6. Use the linear approximation and the original cost function to fill in the chart.

x	C(x)	L(x)	Error C(x) – L(x)
98			
99			
100	$990	$990	0
101	$990.10107	$990.10	.00107
102			
103			
104			

7. **DIFFERENTIALS:**

If y = f(x) is differentiable , then the differential dx is an independent variable and can be given the value of any real number.

The differential dy is defined in terms of dx

dy = f'(x)dx and thus depends on the values of x and dx.

A geometric interpretation is illustrated on **page 251 figure 6.**

Consider a point **P (x, f(x))** on the curve y = f(x) and another point **Q (x + Δx, f(x + Δx)).**

The change in y, $\Delta y = f(x + \Delta x) - f(x)$, while dy represents the change in the linearization (how much the tangent line at x rises or falls).

The smaller Δx, then the closer dy is to Δy.

8. **APPLICATION:**

The radius of a circular disk is given as 24 cm. The maximum error in measurement is 0.2cm.

A. What is the maximum error in the area of the disk calculated with the given radius measurement of 24 cm?

B. What is the relative error?

What is the percentage error?