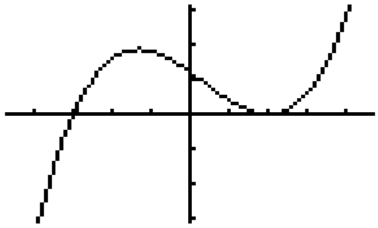
MA 181 SECTION 4.2: MAXIMUM & MINIMUM VALUES

1. LOCAL/RELATIVE EXTREMA:

A function f has a local minimum at x = c, if f(c) is the minimum value of f for some interval containing c.

A function f has a local maximum at x = d, if f(d) is the maximum value of f for some interval containing d.

For example: $f(x) = .1(x - 2)^2(x + 3)$ has a local minimum at x = 2 and a local maximum at x = -4/3.



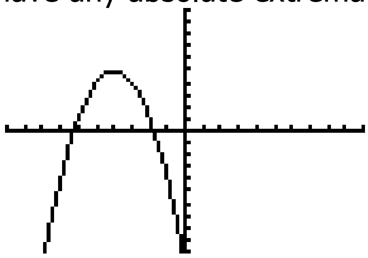
2. ABSOLUTE/GLOBAL EXTREMA:

A function f has an absolute minimum at x = c, if f(c) is the minimum value for f throughout the entire domain of f.

A function f has an absolute maximum at x = d, if f(d) is the maximum value for f throughout the entire domain of f.

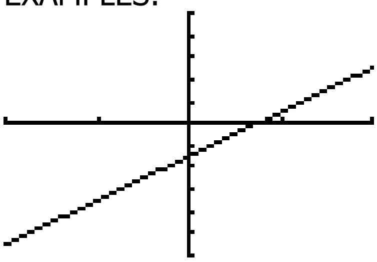
Note: The function $f(x) = .1(x - 2)^2(x + 3)$ does not have any absolute extrema.

$g(x) = -(x + 4)^2 + 5$ has a local and absolute extrema at x = -4.



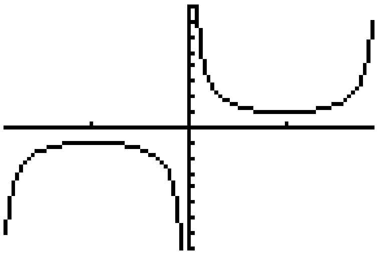
3. THE EXTREME VALUE THEOREM (EVT) If f is a continuous function on [a, b], then f attains an absolute maximum value f(c) and an absolute minimum value f(d) at some numbers c and d in [a, b].

EXAMPLES:

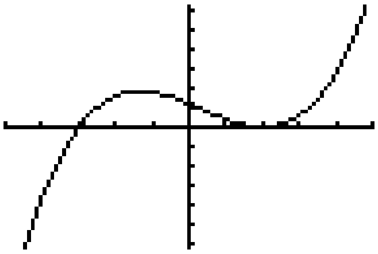


f(x) = 2x – 1.5 on [-2, 2]. Obviously f(x) is continuous on [-2, 2]. f has no local extrema, but has an absolute minimum at x = -2 and an absolute maximum at x = 2.

3. Continued



g(x) = csc x is not continuous on [-3, 3].
g(x) has a vertical asymptote at x = 0.
g(x) has local extrema, but no absolute extrema – since the hypothesis of the EVT is not satisfied.



$f(x) = .1(x - 2)^2(x + 3)$ on [-5, 5] has a local minimum at x = 2 and a local maximum at x = -4/3 and an absolute minimum at x = -5 and an absolute maximum at x = 5.

4. FEMAT'S THEOREM: If f has a local maximum or minimum at x = c, and if f′(c) exists then f′(c) = 0.

5. CRITICAL NUMBER: A critical number c of a function f is a number in the domain of f such that either f′(c) = 0 or f′(c) does not exist.

Note: The existence of a critical number does not imply the existence of a local extrema. However, since a local extrema can only occur at a critical number – we only need to check the critical numbers to determine if the function has any local extrema.

6. CLOSED INTERVAL METHOD: *Outlined on page 266*

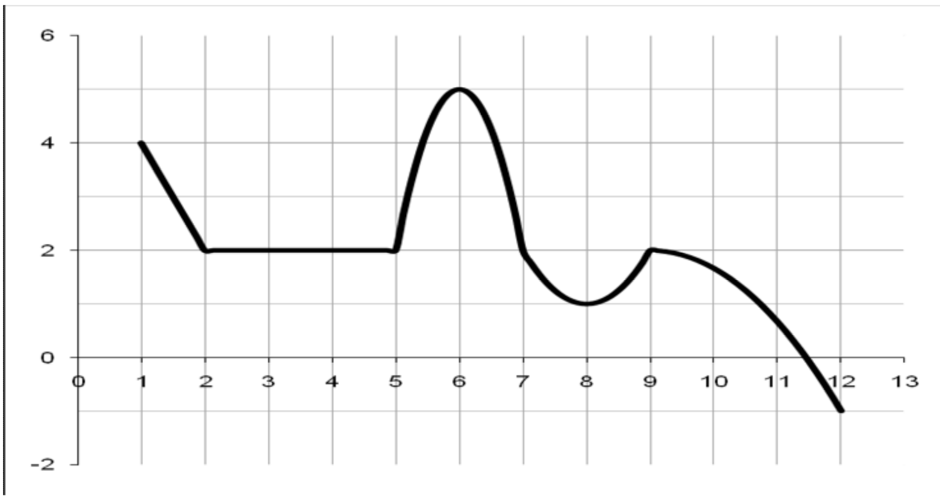
Find the absolute extrema of $f(x) = x^3 - 3x + 1$ on [0, 3]

7. Is the following true:
$H(x) = x^3$
$H'(x) = 3x^2$ ➔ $H'(x) = 0$ for $x = 0$ ➔ So H has a local extrema at $x = 0$.

If it is true, find the local extreme value of H and state whether it is a local minimum or a local maximum.

If it is false, explain why Fermat's theorem does not apply.

8. For each of the numbers 1,2, 4, 5, 6, 8, 9, and 12 state whether the function whose graph is shown has an absolute maximum or minimum, a local maximum or minimum, or neither a maximum nor a minimum.



9. Draw a graph with no local maximum or local minimum but with critical numbers at x = 2 and 4.