MA 181 SECTION 4.3: DERIVATIVES AND THE SHAPES OF CURVES

1. **The Increasing/Decreasing Test:** What does f' say about f?

   A. If f'(x) > 0 on an interval, then f is increasing on that interval.

   B. If f'(x) < 0 on an interval, then f is decreasing on that interval.

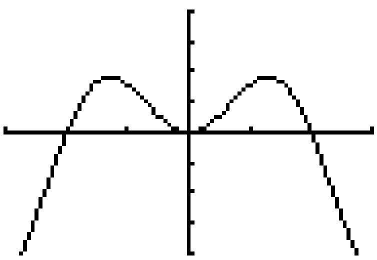
2. **The First Derivative Test:** f is a continuous function and c is a critical number of f.

   A. If f' changes sign from positive to negative at c (*translation: f changes from increasing to decreasing*), then f has a local maximum at c.

   B. If f' changes sign from negative to positive at c (*translation:*

   C. If f' does not change sign at c. That is f' is either positive for x > c and x < c OR f' is negative for x > c and x < c. (Recall c is a critical number of f. Either f'(c) = 0 or f'(c) dne.) Then f has neither a local maximum or minimum at c.

3. The following is a graph of k'(x).



   A. List the critical numbers of k(x).

   B. Where does k have a local minimum?

   C. Where does k have a local maximum?

   D. Does k have any critical numbers at which k has neither a local max nor min?

4. **The Concavity Test:** What does f" say about f?

   A. If f"(x) > 0 on some interval, then f is concave up on that interval.

   B. If f"(x) < 0 on some interval, then f is concave down on that interval.

Note: You can also use the graph of f' to determine the concavity of f.

   C. If f' is increasing on an interval (thus f" > 0 on that interval) f is concave up on that interval.

   D. If f' is decreasing on an interval (thus f" < 0 on that interval) f is concave down on that interval.

5. **The Second Derivative Test:** f" must be continuous near c.

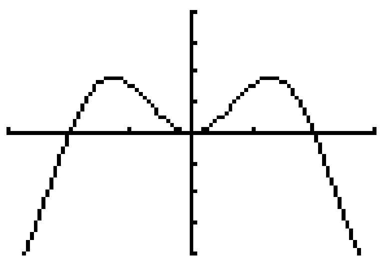
   A. If f'(c) = 0 and f"(c) > 0, then f has local minimum at c.

   B. If f'(c) = 0 and f"(c) < 0, then f has a local maximum at c.

6. **Inflection points:** An inflection point of f(x) occurs when f changes concavity.

   To find inflection points. Find the hypercritical numbers of f (I.e. the critical numbers of f'), then f have an inflection point at a hypercritical number if f" changes sign at that number

7. The following is a graph of k'(x).



   A. List the hypercritical points of k.

   B. On what intervals is k(x) concave up?

   C. On what intervals is k(x) concave down?

   D. Where does k have inflection points?

8. **THE MEAN-VALUE THEOREM (MVT):**

If f is a differentiable function on [a, b], then there exists a number c, a $\leq$ c $\leq$ b such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$. *See figures on page 272.*

9. Application of the MVT: At 2:00 P.M. a car's speedometer reads 30 mph. At 2:10 P.M. it reads 50 mph. Show that at some time between 2 & 2:10 P.M. that the car's acceleration is 120 mph/hr.

10. **USING THE BEST OF CALCULUS AND GRAPHING CALCULATORS TO ANALYZE AND GRAPH A FUNCTION:**

Construct a graph of $f(X) = x^2 e^{-5x}$ that show all of its extrema and concavity.

A. Find the critical numbers of f(x).

B. Does f have any local extrema? Use the Second Derivative Test

C. Does f have any inflection points?

D. Does f have any absolute extrema?

CLASSWORK: Find and Identify Extrema, sketch shape (by hand) w/ aid of calculator.

1. $g(x) = x^4 - 2x^3 + 0.9999x^2$

2. $h(x) = \dfrac{x}{(x - 7)^2} + 15$