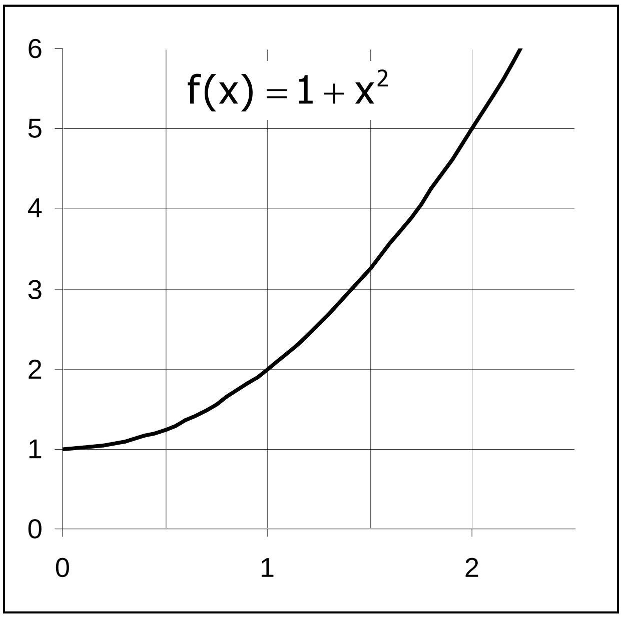
MA 181 SECTION 5.1: THE AREA PROBLEM:



Find the area of the region enclosed by the curve y = f(x) from a to b.

Let $f(x) = 1 + x^2$ on the interval [0, 2]. We want to compute A = the area of the region bounded by $f(x) = 1 + x^2$ on [0, 2].

The shape of this region is not a standard one for which we have a know formula for computing the area.

1. We can estimate the area of the region using rectangles. Draw 4 rectangles of width ½, starting at 0 and ending at 2. Let the height of the rectangles be f(0), f(½), f(1), f(3/2) respectively.

   Compute the area of these 4 rectangles and sum them. This is called the Left-hand sum L_4 – the 4 is because we took 4 subintervals (rectangles). Obviously, this sum is an underestimate of the area.

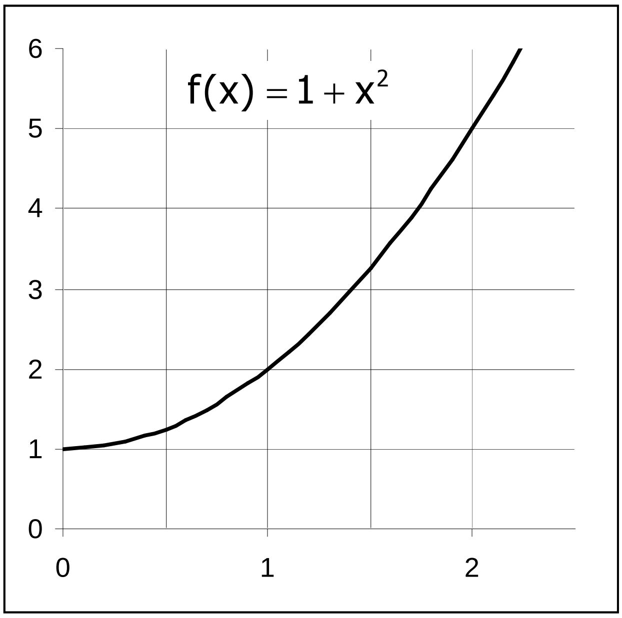
   L_4 = ___________________________

2. Estimate the area of the region by computing the Right-hand Sum (R_4). Draw 4 rectangles of width ½, starting at 0 and ending at 2. Let the height of the rectangles be f(1/2), f(1), f(3/2) and f(1) respectively.

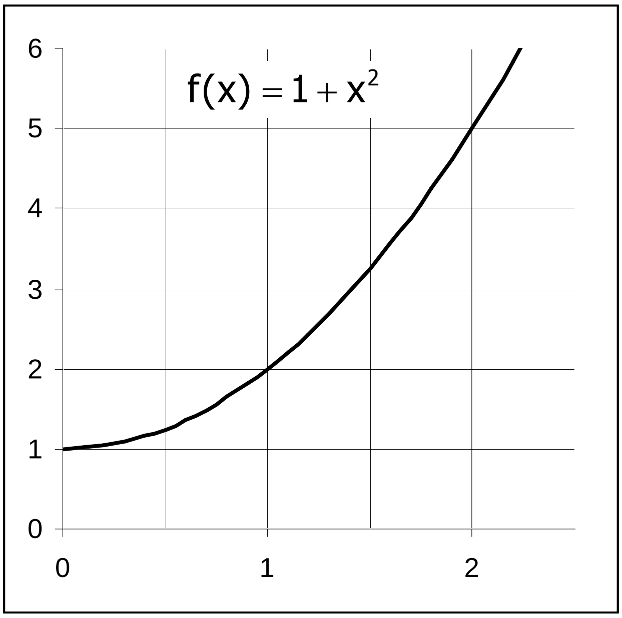
   Compute the area of these 4 rectangles and sum them.

   R_4 = ________________________

   Obviously this is an overestimate of the area of the region.



3. Estimate the area of the region by computing the Midpoint Sum (M_4). Draw 4 rectangles of width ½, starting at 0 and ending at 2. Let the height of the rectangles be f(1/4), f(3/4), f(3/2) and f(7/4) respectively.



Compute the area of these 4 rectangles and sum them.

M_4 = ____________________

Is this estimate an overestimate or an underestimate of the area of the region?

4. Do you think our estimates will become more accurate if we increased the number of rectangles we used to compute L, R, or M?

As the number of rectangles (sub-intervals) n approaches infinity, L_n, R_n, and M_n become better and better approximations for the area A of the region. In fact, we define the area A of the region that lies under the graph of a continuous f is the limit of the sum of areas of approximating rectangles A = $\lim_{n \to \infty} L_n = \lim_{n \to \infty} M_n = \lim_{n \to \infty} R_n$.

We'll use this definition to compute the area of the region bounded by $f(x) = 1 + x^2$ on [0, 2]. $R_n = [f(x_1)\Delta x + f(x_1))\Delta x + . . . + f(x_n)\ \Delta x]$

We are computing the area under the curve y = f(x) on [a, b]. Divide [a, b] into n equal sub-intervals. The width of each subinterval $\Delta x = \dfrac{b - a}{n}$.

In our case a = _________ b = _______ so Δx = _________.

x_1 = _________

x_2 = _________

.

.

x_n = _________

R_n = _________

A = $\lim_{n \to \infty} R_n = \lim_{n \to \infty}$ ______________________

(Problem #17 is similar to this. We will discuss the limits and summation notation later.)

Use NUMERINT to approximate the area under the curve using n = 10, 25, 50, 100. Guess the actual area.