MA 181 SECTION 5.2: THE DEFINITE INTEGRAL

1. The Definition of a Definite Integral is given on page 343.

2. Things to note about a definite integral.

   A. $\int_a^b f(x)\ dx$ is the definite integral of f from a to b.

   B. $\int_a^b f(x)\ dx$ is a numerical value.

   C. $\int$ is the integral symbol.

   D. f(x) is called the integrand.

   E. a & b are the limits of integration.

   F. The process is of computing the numerical value is called integration.

3. In Section 5.1, we approximated the value of definite integrals by using Reimann Sums. In our examples, the graphs of the functions were always positive. The process of computing a definite integral yields the area of the region enclosed by the graph and the x-axis. If the graph is negative (or below the x-axis) then the area is computed as a negative value.

$$\int_0^{2\pi} \sin\ x\ dx = \lim_{n \to \infty} \sum_{n=1}^{\infty} \sin\ (x_i)\ \Delta x$$

Compute R_4

4. We are working towards an efficient process for computing definite integrals. But first, we are going to explore the theory behind this efficient process. So far, we have approximated a definite integral by using Riemann sums. The midpoint rule provides a better approximation to the integral than the using the left or right endpoints of the subintervals to approximate the integral. There are more rules (trapezoid and Simpson's) that give even better approximations. However, we still compute just an approximation when we use a finite Riemann sum to calculate a definite integral. The definition of the definite Integral (p. 354) tells us that the exact value of a definite integral is a limit of Riemann sums. That means by evaluating the appropriate limit, we find the value of a Riemann sum with n subintervals, where n approaches infinity.

5. Use the definition of the definite integral to find $\int_0^2 (1 + x^2)\ dx$.

6. **EXAMPLE 2 ON PAGES 346 – 347 for a similar example.**

7. PROPERTIES OF THE DEFINITE INTEGRAL p. 350 – 352

8. EXAMPLES

A. $\int_{1}^{1} x^2 \cos x \ dx$

B. If $\int_{1}^{5} f(x)\ dx = 12$ and $\int_{4}^{5} f(x)\ dx = 3.6$, compute $\int_{1}^{4} f(x)\ dx$.

C. Find $\int_{0}^{5} f(x)\ dx$ if $f(x) = \begin{cases} 3 & \text{for } x < 3 \\ x & \text{for } x \geq 3 \end{cases}$

D. Find $\int_{-1}^{3} (3 - 2x)\ dx$