MA 181 SECTION 5.3: THE FUNDAMENTAL THEOREM OF CALCULUS

"THE SHORT-CUT TO COMPUTING A DEFINITE INTEGRAL"

1. **Evaluation Theorem:** If f is a continuous function on the interval [a, b], then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where F is any antiderivative of f. (F′ = f)

Calculator, MATH menu, fnint(y_i , x, , a, b)

2. EXAMPLES: "Memorize" all but last two of table on page 358.

A. $\int_0^{2\pi} \sin x\,dx$

B. $\int_1^2 \frac{4 + w^2}{w^3}\,dw$

C. $\int_0^3 |3x - 5|\,dx$

D. $\left|\int_0^3 (3x - 5)dx\right|$

3. The Indefinite Integral, distinguishing between a definite and indefinite integral.

- $\int_a^b f(x)\,dx$ is a number (that gives the area under the graph of f(x)).
- $\int f(x)\,dx$ is a function that is an antiderivative for f(x).

For example: $\int_0^{\pi} \sin x\,dx = 1$ while $\int \sin x\,dx = -\cos x + C$

4. **EXAMPLES:**

A. $\int (10 - \cos x)dx$

B. $\int x^2(1 - x^6)dx$

5. **Net Change Theorem:** The integral of a rate of change is the total change

$$\int_a^b F'(x)dx = F(a) - F(b)$$

Some general descriptions of the use of this theorem in applications can be found on pages 360– 362.

6. **"Real World" connections:**

A. In Section 5.1, we examined the graph of the velocity of an object (that was always moving in the positive direction) and noted that the area under the graph of the velocity function for an object was equal to the distance traveled by the object.

$\int_a^b v(t)\,dt = s(b) - s(a)$, where s is the position function of the object.

B. If s′(t) is the velocity of a car traveling in a positive direction for time t in hours, what is $\int_{2}^{5} s'(t)\,dt$?

C. If the population of a city increases at the rate of r′(t) people per year, what does $\int_{0}^{10} r'(t)\,dt$ represent?

D. If f(x) is the slope of a trail at a distance of x miles from the start of the trail, what does $\int_{3}^{5} f(x)\,dx$ represent?

E. If the units for x are feet and the units for a(x) are pounds per foot, what are the units for da/dx? What are the units for $\int_{2}^{8} a(x)\,dx$?

7. MORE EXAMPLES:

A. An animal population is increasing at a rate of 200 + 50 t per year (where t is measured in years). By how much does the animal population increase between the fourth and tenth year?

B. a(t) = 2t + 3 is the acceleration function (in m/s^2) and the initial velocity is v(0) = -4

i. Find the velocity at time t

ii. Find the distance traveled from $0 \leq t \leq 3$ seconds.

8. **THE EVALUATION THEOREM – MISC. NOTES**

Each of the following definite integrals can't be evaluated. Choose one (or more if it applies) of the following reasons to state why the definite integral can't be evaluated.

CHOICES:

A. We don't know how to find an antiderivative function for the integrand.

B. The integrand is not a continuous function on the interval [a, b].

C. The integral is not a definite integral.

i. $\int_{-1}^{1} x^{-2}\ dx$

ii. $\int \left(1 + x^2\right)\ dx$

iii. $\int_{3}^{5} \sin\ (x^4)\ dx$

iv. $\int \left(2 \sin x \cos x + 2 \cos x\right)\ dx$

9. Compute

A. $\frac{d}{dx}\left[(\sin x + 1)^2\right]$

B. $\frac{d}{dx}\left[\sin^2 x + 2 \sin x\right]$

10. Compare your answers in 9A & 9B. Are they the same?

11. Can you use #9 to find $\int (2 \sin x \cos x + 2 \cos x)\ dx$? If so write the answer(s) here.