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TEST POLICIES (REMINDERS)

If you know that you will be absent on the day of a test, it may be possible to make arrangements with me to take the test on an earlier day. This alternative is not automatic!! Each case will be considered individually. You should notify me as soon as possible regarding planned absences. There are no make-up exams provided for unexpected absences. If you miss an exam for an unexpected absence, the average that you earn on the final will make-up the score for your missed exam. There are no exceptions – do not ask!! When you complete the test, hand it to me personally. You may leave the room at this time. The only questions permitted during an exam are in reference to a misprint, omission, or illegible text. You are responsible for being prepared for the exam. Do not ask me how to do the problem, ask for a hint about how to do the problem, or ask whether or not your answer is correct. Please do not share your misery of being absent for a particular topic, that you forgot how to do the problem, or that you do not have enough time. You will not have the time to figure out problems on exam day. You are to come prepared, polished, and ready to complete the exam. You must turn in your test paper when time is called. I will give a five minute warning before collecting exams. If you do not hand in your paper at collection time, I will not accept it later. It is your responsibility to keep track of time during the exam.

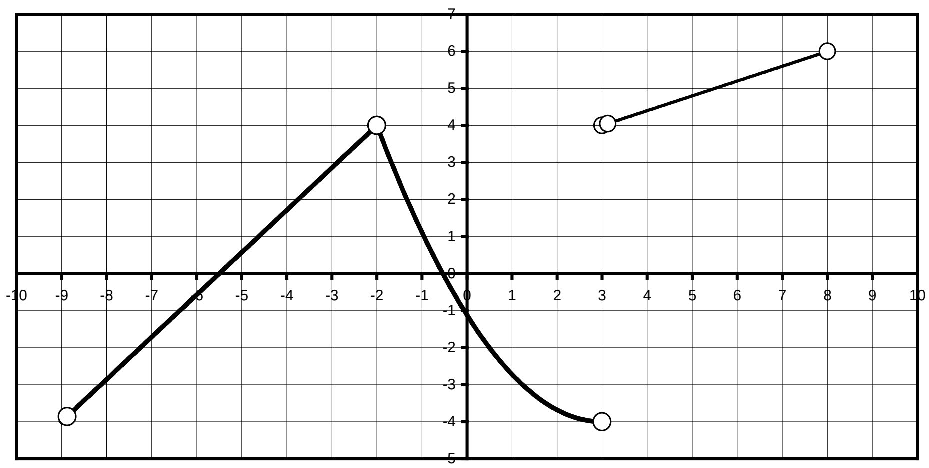
**MY HONOR POLICY:**

(1)Be prepared to move to a designated seat if requested.
(2)You are not to leave the room without permission.
(3)You must not look anywhere in the room other than at your own test paper.
(4)You may not use or even touch a cell phone. Remember your cell phone is to be silenced in class.
(5)You may not speak to another student.
(6)You may not share materials with another student.
(7)Have all your materials ready. You may not retrieve items from your backpack etc.

**Failure to observe all of the policies will result in a zero score for the test or quiz.**

This review sheet contains review problems and answers. Also review group worksheets.

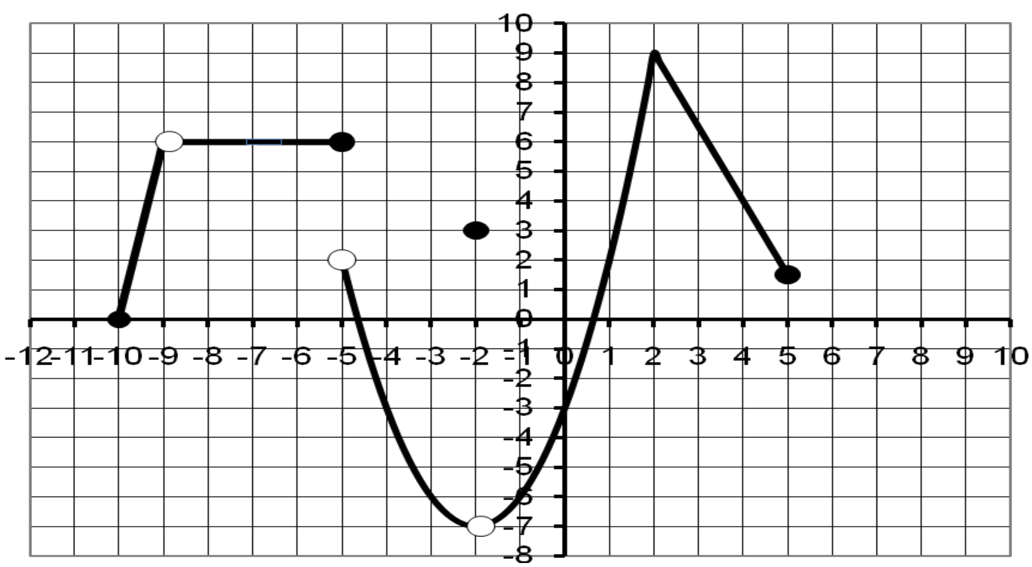
1. Use the graph of f below to answer the following questions:



A. $\lim_{x \to 3^+} f(x)$ B. $\lim_{x \to 3^-} f(x)$ C. $\lim_{x \to 3} f(x)$

D. f(3) E. $\lim_{x \to -2} f(x)$ F. f(-2)

2. Use the graph of f below to answer the following questions:



A. $\lim_{x \to -9^+} f(x)$ B. $\lim_{x \to -9^-} f(x)$ C. $\lim_{x \to -9} f(x)$ D. f(-9)

E. $\lim_{x \to -5^+} f(x)$ F. $\lim_{x \to -5^-} f(x)$ G. $\lim_{x \to -5} f(x)$ H. f(-5)

I. $\lim_{x \to -2} f(x)$ J. f(-2) K. $\lim_{x \to 2} f(x)$ L. f(2)

3. Use algebra to find the value of c so that

A. $\lim_{x \to 3} \frac{(x - 3)(x + c)}{x^2 - 4x + 3} = 5$ B. $\lim_{x \to 4} \frac{(x - 4)(x + c)}{x^2 - 3x - 4} = \frac{2}{5}$

4. Find the following limits. You must be able to support your answer with an explanation

such as

1. The function is continuous, so direct substitution is possible.
2. There is a removable discontinuity at $x = a$.
3. The limit is infinity (or negative infinity) because there is a vertical asymptote at $x = a$.
4. The limit does not exist because the limit is only one-sided.
5. For limits at infinity,
    a. The limit exists because the function has a horizontal asymptote of y = ___.
    b. The limit is infinity (or negative infinity) because the function does not have a horizontal asymptote.
6. The limit does not exist because the limit from the left and the limit from the right are not the same. There is a jump discontinuity

A. $\lim\limits_{x \to 1} (x^{17} - x + 3)$ B. $\lim\limits_{x \to 1} \dfrac{x - 1}{\sqrt{x} - 1}$ C. $\lim\limits_{x \to \infty} \dfrac{7 + 3x}{4 - x}$

D. $\lim\limits_{x \to \infty} \sqrt{\dfrac{x + 8x^2}{2x^2 - 1}}$ E. $\lim\limits_{x \to -\infty} \dfrac{x^7 - 1}{x^6 + 1}$ F. $\lim\limits_{x \to \infty} \left(\sqrt{x^2 + x} - x\right)$

G. $\lim\limits_{x \to 2} \dfrac{x - 2}{x + 2}$ H. $\lim\limits_{x \to -\infty} \dfrac{7 + 3x}{4 - x}$ I. $\lim\limits_{x \to \infty} \dfrac{5x^2}{3x + 1}$

J. $\lim\limits_{x \to \infty} \sqrt[4]{\dfrac{16x}{x - 7}}$ K. $\lim\limits_{x \to \infty} \dfrac{4x^3 + 9x}{7x^3 - 5x^2}$ L. $\lim\limits_{x \to 3} \dfrac{x^2 - 4}{x - 3}$

M. $\lim\limits_{x \to -7} \dfrac{x^2 - 49}{x + 7}$ N. $\lim\limits_{x \to -7} \dfrac{x + 5}{x^2 - 49}$ O. $\lim\limits_{x \to \infty} \dfrac{x - 1}{x - \sqrt{x}}$

5. A. At what value of x does the function $f(x) = \dfrac{(x + 1)^2}{x^2 - 1}$ have a removable discontinuity?

   B. At what value of x does the function $g(x) = \dfrac{x^2 - 9}{(x - 3)(x + 4)}$ have a removable discontinuity?

6. A. At what value of x does the function $f(x) = \dfrac{(x + 1)^2}{x^2 - 1}$ have an infinite discontinuity?

   B. At what value of x does the function $g(x) = \dfrac{x^2 - 9}{(x - 3)(x + 4)}$ have an infinite discontinuity?

7. A. At what value of x does the function $g(x) = \dfrac{|x + 3|}{x + 3}$ have a jump discontinuity?

   B. At what value of x does the function $h(x) = \begin{cases} x^2 & x \le 0 \\ x^2 + 5 & x > 0 \end{cases}$ have a jump discontinuity?

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8. Find a and b so that f is continuous everywhere.

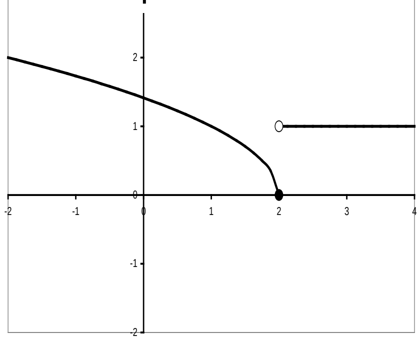
$$f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2} & \text{if } x < 2 \\ ax^2 - bx + 3 & \text{if } 2 \le x < 3 \\ 2x - a + b & \text{if } x \ge 3 \end{cases}$$

9. Sketch a single graph of f(x) has all of the following properties:

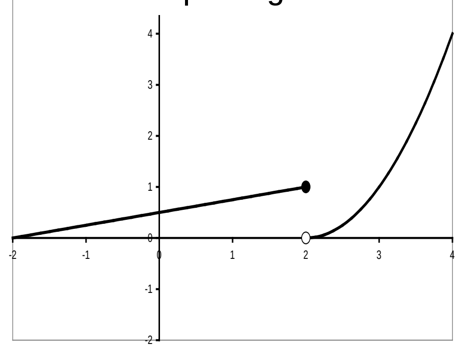
$$\lim_{x \to -1^-} f(x) = \infty, \quad \lim_{x \to -1^+} f(x) = -\infty, \quad \lim_{x \to \infty} f(x) = \lim_{x \to -\infty} f(x) = 1$$

10. Given the functions f and g (defined graphically) and h, j and k (defined algebraically), compute each of the following limits, or state why they do not exist.

Graph of f



Graph of g



$$h(x) = \frac{x^2 - 4}{x - 2}$$

$$j(x) = \begin{cases} 1 & \text{if } x < 2 \\ 0 & \text{if } x \ge 2 \end{cases}$$

$$k(x) = x^2$$

SHOW STEPS!! Tell whether or not the limit laws can be applied. Use one-sided limit laws as necessary.

A. $\lim_{x \to 2} \left[h(x) + g(x)\right]$

B. $\lim_{x \to 2} \left[f(x) \cdot k(x)\right]$

C. $\lim_{x \to 2} \left[f(x) \bullet j(x)\right]$

D. $\lim_{x \to 2} \left[h(x) + k(x)\right]$

11. A. Find the value x at which the curve $y = \dfrac{x^2 - 16}{x^2 - 5x + 4}$ has a vertical asymptote.

B. Find the value y at which the curve $y = \dfrac{x^2 - 16}{x^2 - 5x + 4}$ has a horizontal asymptote.

12. Given that $\lim_{x \to 3} f(x) = 5$, $\lim_{x \to 3} g(x) = 0$, and $\lim_{x \to 3} h(x) = -8$, find the following limits, if they exist. If a limit does not exist, explain why.

A. $\lim_{x \to 3} (f(x) + h(x))$

B. $\lim_{x \to 3} x^2 f(x)$

C. $\lim_{x \to 3} f^2(X)$

D. $\lim_{x \to 3} \dfrac{f(x)}{2h(x)}$

E. $\lim_{x \to 3} \dfrac{g(x)}{f(x)}$

F. $\lim_{x \to 3} \dfrac{f(x)}{g(x)}$

G. $\lim_{x \to 3} \dfrac{2h(x)}{f(x) - h(x)}$

H. $\lim_{x \to 3} \sqrt[3]{h(x)}$

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13. Find an equation of the line tangent to $f(x) = x^2 - 4x$ at the point (6, 12).

14. Suppose that the line tangent to the graph of $y = f(x)$ at $x = 3$ passes through the points (-2, 3) and (4, -1).
    A. Find $f'(3)$.
    B. Find the equation of the line tangent to $f(x)$ at $x = 3$.
    C. Find $f(3)$.

15. Below are the water levels for the Red River recorded in Fargo, North Dakota during the flood of 1997. Readings were made at 1:00 A.M. each day.

| **Date** | 04/01 | 04/02 | 04/03 | 04/04 | 04/05 | 04/06 |
|---|---|---|---|---|---|---|
| **Level** (ft) | 15.62 | 16.30 | 17.21 | 20.25 | 23.63 | 27.47 |

    A. Find the average rate of change of the water level over the following periods.
    [04/04, 04/05]     [04/03, 04/04]

    B. Estimate the rate of change of water level on April 4th.

16. Let $s(t) = 4t^3 - 24t^2 + 36t$ be the distance function for a particle, where t is measured in seconds, and s is in feet. Include units with your answers.

    A. Find the position of the particle at $t = 2$ and $t = 6$.

    B. Find the average velocity of the particle over [2,4].

17. If $f(x) = 3x^2 + 5x$, find $f'(1)$ by simplifying the appropriate limit of difference quotient.

    Hint: $f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$

18. Let W(t) be the weight of a child (in pounds) who is t years old. Interpret the following in practical terms.

    A. $W(4) = 42$     B. $W'(2) = 10$

19. Use the limit of the difference quotient to derive $f'(a)$

    A. $f(x) = \sqrt{3x - 2}$     B. $f(x) = \frac{x + 1}{2x - 1}$

    C. $f(x) = 3x^2 - 14x$     D. $f(x) = \frac{4}{\sqrt{1 - x}}$

**ANSWERS:**

1. A. 4   B. -4   C. dne
   D. dne   E. 4   F. dne

2. A. 6   B. 6   C. 6   D. dne
   E. 2   F. 6   G. dne   H. 6
   I. -7   J. 3   K. 9   L. 9

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3. A. $c = 7$ B. $c = -2$

4.
A. 3 by direct sub. this function is continuous
B. 2 (by factoring)
C. -3, this function has a H.A. of $y = -3$
D. 2, Applied Limit Law and noted that the quotient function has a H.A. of $y = 4$.
E. $-\infty$, this function has an oblique asymptote
F. 1/2, (by rationalizing numerator & example 10 technique on page 137)
G. 0, direct substitution
H. -3, horizontal asymptote
I. $\infty$, no horizontal asymptote
J. 2, horizontal asymptote
K. 4/7, horizontal asymptote
L. dne, vertical asymptote
M. -14, removable discontinuity
N. dne, vertical asymptote
O. 1, horizontal asymptote

5. A. $x = -1$ B. $x = 3$

6. A. $x = 1$ B. $x = -4$

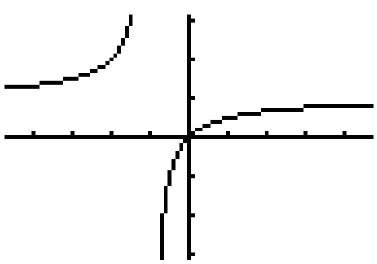
7. A. $x = -3$ B. $x = 0$

8. Using the first piece, decide that when $x = 2$, $ax^2 - bx + 3$ should be 4. If $x = 2$, then $ax^2 - bx + 3 = 4a - 2b + 3$, thus $4a - 2b + 3 = 4 \Rightarrow 4a - 2b = 1$.

Using the third piece, decide that when $x = 2$, $ax^2 - bx + 3$ should be $2(3) - a + b$. If $x = 3$, then $ax^2 - bx + 3 = 6a - 3b + 3$, thus $9a - 3b + 3 = 6 - a + b \Rightarrow 10a - 4b = 3$.

Solving the two results simultaneously gives $a = b = ½$.

9.



10. A. dne, unequal one-sided limits B. dne, unequal one-sided limits
C. 0 by one-sided limit laws D. 8, by limit laws

11. A. $x = 1$ B. $y = 1$

12. A. -3 B. 45 c. 25
D. -5/16 E. 0 F. dne
G. -16/13 H. -2

13. $y = 8(x - 6) + 12 \Rightarrow y = 8x - 36$

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14. A. $-\frac{2}{3}$ B. $y = -\frac{2}{3}x + \frac{5}{3}$ or $y = -\frac{2}{3}(x-3) - \frac{1}{3}$ C. $-\frac{1}{3}$

15. A. 3.38 ft/day, 3.04 ft/day B. about 3.21 ft/day

16. A. 8 ft., 216 ft. B. 4ft/s 17. 11

18. A. At age 4 years, the child weighs 42 lbs.

B. At age 2 years, the child gains 10 lbs per year.

19. A. $f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h\to 0} \frac{\sqrt{3\bullet(a+h) - 2} - \sqrt{3a - 2}}{h}$

$$= \lim_{h\to 0} \frac{\sqrt{3\bullet(a+h) - 2} - \sqrt{3a - 2}}{h} \bullet \frac{\sqrt{3\bullet(a+h) - 2} + \sqrt{3a - 2}}{\sqrt{3\bullet(a+h) - 2} + \sqrt{3a - 2}}$$

$$= \lim_{h\to 0} \frac{3(a + h) - 2 - (3a - 2)}{h\left(\sqrt{3\bullet(a+h) - 2} + \sqrt{3a - 2}\right)} = \lim_{h\to 0} \frac{3a + 3h - 2 - 3a + 2}{h\left(\sqrt{3\bullet(a+h) - 2} + \sqrt{3a - 2}\right)}$$

$$= \lim_{h\to 0} \frac{3h}{h\left(\sqrt{3\bullet(a+h) - 2} + \sqrt{3a - 2}\right)} = \lim_{h\to 0} \frac{3}{\left(\sqrt{3\bullet(a+h) - 2} + \sqrt{3a - 2}\right)}$$

$$= \lim_{h\to 0} \frac{3}{\left(\sqrt{3\bullet(a+h) - 2} + \sqrt{3a - 2}\right)} = \frac{3}{2\sqrt{3a - 2}}$$

B. $f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h\to 0} \frac{\frac{(a + h) + 1}{2(a + h) - 1} - \frac{a + 1}{2a - 1}}{h}$

$$= \lim_{h\to 0} \frac{\frac{(a + h) + 1}{2(a + h) - 1} \bullet \frac{2a - 1}{2a - 1} - \frac{a + 1}{2a - 1} \bullet \frac{2(a + h) - 1}{2(a + h) - 1}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{\left[(a + h) + 1\right](2a - 1)}{(2(a + h) - 1)(2a - 1)} - \frac{(a + 1)\left[2(a + h) - 1\right]}{(2(a + h) - 1)(2a - 1)}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{2a^2 + 2ah + 2a - a - h - 1}{(2(a + h) - 1)(2a - 1)} - \frac{2a^2 + 2ah - a + 2a + 2h - 1}{(2(a + h) - 1)(2a - 1)}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{\left(2a^2 + 2ah + 2a - a - h - 1\right) - \left(2a^2 + 2ah - a + 2a + 2h - 1\right)}{(2(a + h) - 1)(2a - 1)}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{3h}{(2(a+h)-1)(2a-1)}}{h} = \lim_{h\to 0} \frac{3}{(2(a+h)-1)(2a-1)} = \frac{3}{(2a-1)^2}$$

C. $$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{3(a+h)^2 - 14(a+h) - \left[3a^2 - 14a\right]}{h}$$

$$= \lim_{h\to 0} \frac{3\left(a^2 + 2ah + h^2\right) - 14(a+h) - \left[3a^2 - 14a\right]}{h}$$

$$= \lim_{h\to 0} \frac{3a^2 + 6ah + 3h^2 - 14a - 14h - 3a^2 + 14a}{h}$$

$$= \lim_{h\to 0} \frac{6ah + 3h^2 - 14h}{h} = \lim_{h\to 0} \frac{h(6a + 3h - 14)}{h} = \lim_{h\to 0} (6a + 3h - 14) = 6a - 14$$

D. $$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{\frac{4}{\sqrt{1-(a+h)}} - \frac{4}{\sqrt{1-a}}}{h}$$

$$= 4\lim_{h\to 0} \frac{\frac{\sqrt{1-a} - \sqrt{1-(a+h)}}{\sqrt{1-(a+h)}\bullet\sqrt{1-a}}}{h} = 4\lim_{h\to 0} \frac{\sqrt{1-a} - \sqrt{1-(a+h)}}{h\left(\sqrt{1-(a+h)}\bullet\sqrt{1-a}\right)}$$

$$= 4\lim_{h\to 0} \frac{\sqrt{1-a} - \sqrt{1-(a+h)}}{h\left(\sqrt{1-(a+h)}\bullet\sqrt{1-a}\right)} \bullet \frac{\sqrt{1-a} + \sqrt{1-(a+h)}}{\sqrt{1-a} + \sqrt{1-(a+h)}}$$

$$= 4\lim_{h\to 0} \frac{(1-a) - (1-(a+h))}{h\left(\sqrt{1-(a+h)}\bullet\sqrt{1-a}\right)\left[\sqrt{1-a} + \sqrt{1-(a+h)}\right]}$$

$$= 4\lim_{h\to 0} \frac{-h}{h\left(\sqrt{1-(a+h)}\bullet\sqrt{1-a}\right)\left[\sqrt{1-a} + \sqrt{1-(a+h)}\right]}$$

$$= 4\lim_{h\to 0} \frac{-1}{\left(\sqrt{1-(a+h)}\bullet\sqrt{1-a}\right)\left[\sqrt{1-a} + \sqrt{1-(a+h)}\right]}$$

$$= \frac{-4}{\left(\sqrt{1-a}\bullet\sqrt{1-a}\right)\left[\sqrt{1-a} + \sqrt{1-a}\right]} = \frac{-4}{(1-a)\bullet 2\sqrt{1-a}} = \frac{-2}{(1-a)^{\frac{3}{2}}}$$