TEST POLICIES (REMINDERS)

If you know that you will be absent on the day of a test, it may be possible to make arrangements with me to take the test on an earlier day. This alternative is not automatic!! Each case will be considered individually. You should notify me as soon as possible regarding planned absences. There are no make-up exams provided for unexpected absences. If you miss an exam for an unexpected absence, the average that you earn on the final will make-up the score for your missed exam. There are no exceptions – do not ask!! When you complete the test, hand it to me personally. You may leave the room at this time. The only questions permitted during an exam are in reference to a misprint, omission, or illegible text. You are responsible for being prepared for the exam. Do not ask me how to do the problem, ask for a hint about how to do the problem, or ask whether or not your answer is correct. Please do not share your misery of being absent for a particular topic, that you forgot how to do the problem, or that you do not have enough time. You will not have the time to figure out problems on exam day. You are to come prepared, polished, and ready to complete the exam. You must turn in your test paper when time is called. I will give a five minute warning before collecting exams. If you do not hand in your paper at collection time, I will not accept it later. It is your responsibility to keep track of time during the exam.

MY HONOR POLICY:

(1)Be prepared to move to a designated seat if requested.
(2)You are not to leave the room without permission.
(3)You must not look anywhere in the room other than at your own test paper.
(4)You may not use or even touch a cell phone. Remember your cell phone is to be silenced in class.
(5)You may not speak to another student.
(6)You may not share materials with another student.
(7)Have all your materials ready. You may not retrieve items from your backpack etc.

Failure to observe all of the policies will result in a zero score for the test or quiz.

This review sheet contains review problems and answers. You are encouraged to work similar problems from the end of chapter reviews in the textbook. The answers are included at the end of the review.

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1. Find the derivative

A. $y = (\sin x)^x$

B. $y = \ln(3x^2 + 1 + e^{-x})$

C. $y = 2x + \ln x$

D. $y = x \ln(x^2 - 3)$

E. $y = \sqrt{x}\ \ln\ x$

F. $y = x^{x^2}$

G. $y = e^{(2x + 1)} \ln(x^2 + 1)$

H. $y = 5^{\tan x}$

I. $y = \ln\sqrt{\dfrac{3x^2 + 5}{x^3 + 4}}$

J. $y = \ln(x^6 / \cos x)$

K. $y = \pi^e + \pi^x + x^\pi + \left(\pi^e\right)^x + x^{\pi^e} + \tan x + \ln x + \ln \pi$

L. $y = (e^x + \tan x)^{(2x + 7)}$

M. $y = \left(e^x + x^3\right)^{x^4}$

N. $y = \ln(x^4 + \sin x)$

O. $y = \ln\left(x^5 \cos x\right)^8$

P. $y = \ln(x^5 + \tan x)$

Q. $y = \ln\left(\sin x \bullet \cos x\right)^8$

r. $y = x^{\sin x}$

S. $y = \left(\sec x\right)^{x^3}$

2. Suppose that g is differentiable for all x. Find f ' in terms of g and g'.

A. $f(x) = (g(x))^3$

B. $f(x) = x^3 g(\ln x)$

C. $f(x) = \sin^5 (g(x))$

D. $f(x) = \sqrt{\dfrac{g(x)}{2x + 5}}$

3. The equation of motion of a particle is $s = t^3 - 3t + 2$, where s is in meters and $t > 0$ is in seconds. Find (Give units with your answers !!)

A. The position of the particle after 3 seconds.
B. The velocity of the particle after 3 seconds.
C. Is the object ever at rest (is the velocity ever zero)?
D. Find the acceleration of the particle after 3 seconds.
E. Is the particle ever speeding up? If so, when? Justify your answer either way!!

4. Suppose that a baseball is tossed straight upward and that it height (in feet) as a function of time (in seconds) is given by $h(t) = 128t - 16t^2$.

A. Find the average velocity of the ball from t = 1 to t = 4.
B. Find the instantaneous velocity at time t.
C. Find the instantaneous velocity of the ball t = 2.
D. Find the acceleration of the ball at time t.
E. When does the ball reach it maximum height?
F. How long before the ball hits the ground?
G. When is the ball slowing down?
H. At what time is the height of the ball 112 feet?

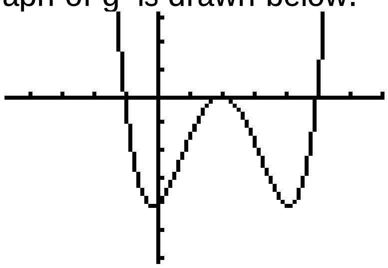
5. Find the equation of the tangent line to the curve $y = (x^2 + 1)^3$ at the point (1, 8).

6. Let f be a function with f(1) = 4 and whose derivative is $f'(x) = \sqrt{x^2 + 1}$.
   - A. Write a linear approximation for f at x = 1.
   - B. Use the linear approximation to estimate f(1.1).
   - C. Is the approximation for f(1.1) greater than or less than the true value of f(1.1)? Explain.

7. Suppose $g''(x) = (x^2 - 25)(x - 3)^2$
   - A. How many DISTINCT zeros does g"(x) have?
   - B. At what values of x do they occur?
   - C. How many inflection points does g(x) have?
   - D. At what values of x do they occur? Explain.

8. Use the 2nd derivative of $f(x) = 2X^3 - 3X^2 - 6X + 100$ to determine where f(x) is concave down.

9. The graph of g' is drawn below.



   - A. List all the critical numbers for g. (Assume g is defined on [-5, 7])
   - B. Classify all the local extrema of g. Explain how you determined your answers.
   - C. Where is g increasing? List the interval(s).
   - D. Where is g decreasing? List the interval(s).
   - E. Where is g concave up? List the interval(s).
   - F. Where is g concave down? List the interval(s).
   - G. How many inflection points does g have? Where are they?

10. At what value of x does $f(x) = x^3 - 3x^2 - 9x$ change from decreasing to increasing? from concave down to concave up? Give exact values not decimal approximations.

11. If $f''(x) = x^3 - 6x^2 + 9x$, how many inflection points does f(x) have? At what values of x do they occur?

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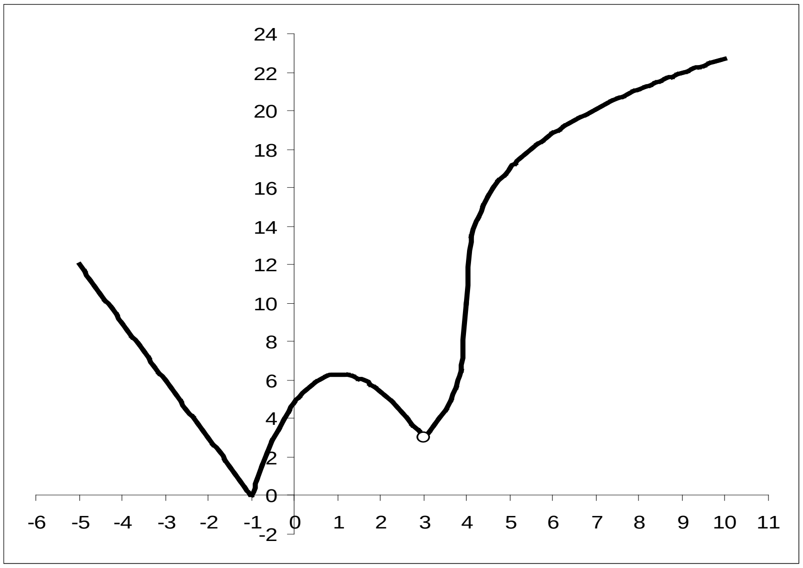
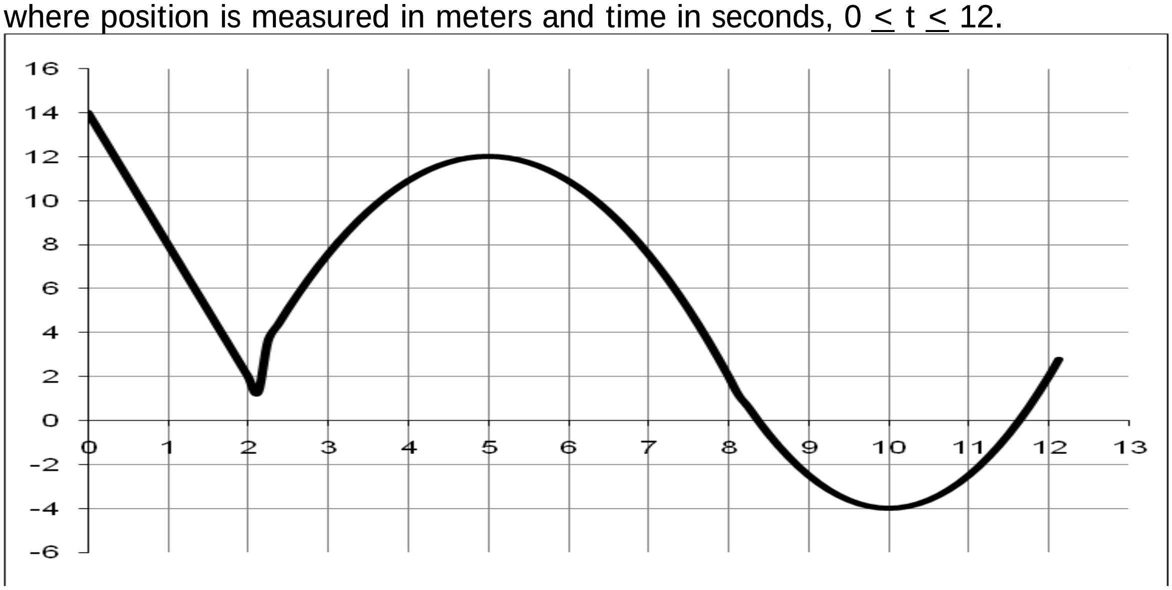
12. Given the graph of y = g'(x), assume that g is defined for all real numbers, answer the following:



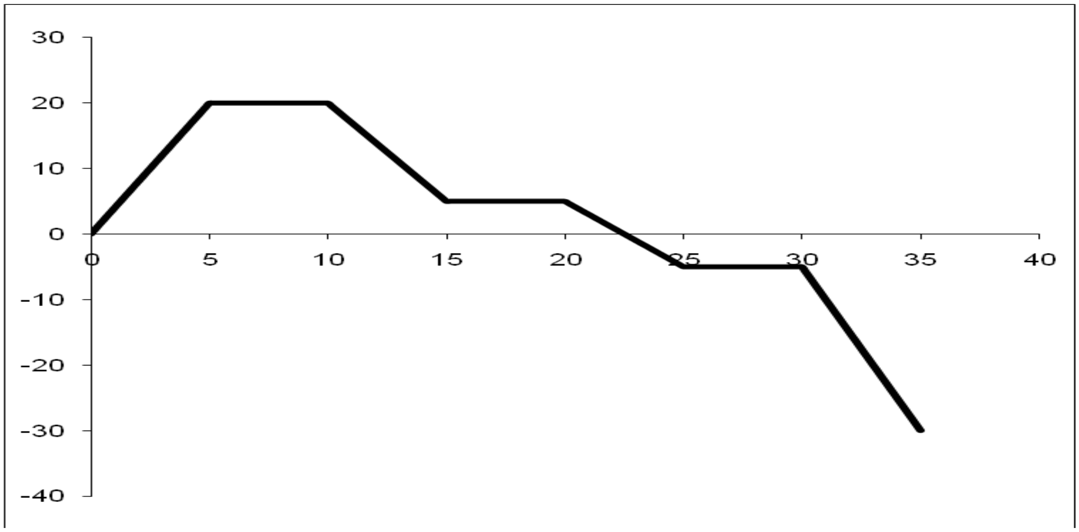
A. List all the critical numbers for g.
B. Classify all the local extrema of g. Explain how you determined your answers.
C. Where is g increasing? List the interval(s).
D. Where is g decreasing? List the interval(s).
E. Where is g concave up? List the interval(s).
F. Where is g concave down? List the interval(s).
G. How many inflection points does g have? Where are they?

13. Suppose the following is the graph of the derivative of a position function, s, for some object where position is measured in meters and time in seconds, $0 \le t \le 12$.



A. What is the name of this function and what are its units?
B. s'(5) = 12. What does this imply about the object?
C. Is the object ever at rest? If so, at what time(s)?
D. At what time(s) is the object speeding up?
E. At what time(s) is the object slowing down?
F. At what time(s) does the object change directions?
G. Is the acceleration of the object ever zero? If so, at what time(s)?
H. Is the acceleration ever constant? If so, at what time(s)?

14. Given the graph of $y = g'(x)$, assume that g is defined on [0, 35].



A. List all the critical numbers for g.
B. Classify all the local extrema of g. Explain how you determined your answers.
C. Where is g increasing? List the interval(s).
D. Where is g decreasing? List the interval(s).

Assume that g' is the velocity function for the position of an object with units given by m/sec.

E. Where is the object speeding up? List the interval(s).
F. Where is the object slowing down? List the interval(s).
G. Where is the object going the fastest?
H. Where is the object going the slowest?

15. Suppose f is a continuous function and that

$f'(x) = (x - 3)(x + 1)^{2/3}$ and $f''(x) = \dfrac{5x - 3}{3(x + 1)^{1/3}}$

A. List the critical numbers of f.
B. On what interval(s) is the function f increasing?
C. On what interval(s) is the function f decreasing?
D. At what value(s) of x, if any, does f have a local maximum?
E. At what value(s) of x, if any, does f have a local minimum?
F. List the hypercritical numbers of f
G. On what interval(s) is the function f concave upward?
H. On what interval(s) is the function f concave downward?
I. At what value(s) of x does f have an inflection point?

16. Suppose g is a continuous function with $g'(x) = (x - 2)^2(x + 5)$ and $g''(x) = (x - 2)(3x + 8)$.

A. List the critical numbers of g.
B. On what interval(s) is the function g increasing?
C. On what interval(s) is the function g decreasing?
D. At what value(s) of x, if any, does g have a local maximum?
E. At what value(s) of x, if any, does g have a local minimum?
F. List the hypercritical numbers of g
G. On what interval(s) is the function g concave upward?
H. At what value(s) of x does g have an inflection point?

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17. A. A large, **closed** shipping container with a **square** base is to be made from 1000 square feet of fiberboard. Find the dimensions of the container with the greatest volume.

B. A rancher wants to fence in a rectangular corral enclosing 1300 square yards and must divide it in half with a fence down the middle. If the perimeter fence costs \$5 per yard and the fence down the middle costs \$3 per yard, determine the dimensions of the corral so that the fencing cost will be minimized.

ANSWERS:

1. A. $y' = (\sin x)^x[\ln(\sin x) + x\cot x]$

B. $y' = \dfrac{6x - e^{-x}}{3x^2 + 1 - e^{-x}}$

C. $y' = 2 + 1/x$

D. $y' = \ln(x^2 - 3) + 2x^2/(x^2 - 3)$

E. $y' = \dfrac{1}{2\sqrt{x}}\ln x + \dfrac{\sqrt{x}}{x}$

F. $y' = x^{x^2}\left[2x\ln x + x\right]$

G. $y' = 2e^{(2x+1)}\left[\ln\left(x^2 + 1\right) + \dfrac{x}{x^2 + 1}\right]$

H. $5^{\tan x}(\sec^2 x)\ln 5$

I. $y' = \dfrac{1}{2}\left[\dfrac{6x}{3x^2 + 5} - \dfrac{3x^2}{x^3 + 4}\right]$

J. $y = 6\ln x - \ln\cos x \Rightarrow y' = 6/x + \tan x$

K. $y' = 0 + (\ln\pi)\pi^x + \pi x^{\pi - 1} + \ln\left(\pi^e\right)\left(\pi^e\right)^x + \pi^e x^{\pi^e - 1} + \sec^2 x + 1/x$

L. $y' = (e^x + \tan x)^{(2x+7)}\left[2\ln\left(e^x + \tan x\right) + (2x + 7)\dfrac{\left(e^x + \sec^2 x\right)}{e^x + \tan x}\right]$

M. $y' = \left(e^x + x^3\right)^{x^4}\left[4x^3\ln\left(e^x + x^3\right) + x^4\dfrac{\left(e^x + 3x^2\right)}{e^x + x^3}\right]$

$= x^3\left(e^x + x^3\right)^{x^4}\left[4\ln\left(e^x + x^3\right) + x\dfrac{\left(e^x + 3x^2\right)}{e^x + x^3}\right]$

N. $y' = \dfrac{4x^3 + \cos x}{x^4 + \sin x}$

O. $y = 8[\ln(x^5) + \ln(\cos x)] = 8[5\ln x + \ln(\cos x)]$, $y' = 8\left[\dfrac{5}{x} + \dfrac{-\sin x}{\cos x}\right] = 8\left[\dfrac{5}{x} - \tan x\right]$

P. $y' = \dfrac{4x^5 + \sec^2 x}{x^5 + \tan x}$

Q. $y = 8[\ln(\sin x) + \ln(\cos x)]$, $y' = 8\left[\dfrac{\cos x}{\sin x} + \dfrac{-\sin x}{\cos x}\right] = 8\left[\cot x - \tan x\right]$

R. $y' = x^{\sin x}\left[\cos x\ln x + \dfrac{\sin x}{x}\right]$

S. $y' = (\sec x)^{x^3}\left[3x^2\ln\sec x + x^3\dfrac{\sec x\tan x}{\sec x}\right]$

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2. A. $f'(x) = 3(g(x))^2 g'(x)$ B. $f'(x) = 3x^2 g(\ln x) + x^2 g'(\ln x)$

C. $f'(x) = 5\sin^4(g(x))\cos((g(x))g'(x)$

D. $$f'(x) = \frac{1}{2}\left(\frac{g(x)}{2x+5}\right)^{\frac{-1}{2}}\left[\frac{g'(x)(2x+5) - g(x)\bullet 2}{(2x+5)^2}\right]$$

3. A. $s(3) = 20$ meters B. $s'(3) = 24$ m/s
C. yes, at time $t = 1$ second D. 18 m/s^2
E. Yes, when $s'(t)$ & $s''(t)$ are the same sign. $s'(t)$ & $s''(t)$ are both > 0 for $t > 1$.
Answer: The particle is speeding up for times $t > 1$ second.

4. A. 48 ft/s B. $h'(t) = 128 - 32t$ C. 64 ft/s
D. -32 ft/s^2 E. $t = 4$ seconds F. 8 seconds
G. When $h'(t)$ and $h''(t)$ are opposite signs. $h''(t) < 0$ on $(0, 8)$, $h'(t) > 0$ on $(0, 4)$
Answer: The ball is slowing down for $0 < t < 4$ seconds.
H. At times $t = 1$ & 7 seconds.

5. $y'(1) = 24$, tangent line is $L(x) = 24(x - 1) + 8$

6. A. $L(X) = 4 + \sqrt{2}(x - 1)$ B. $L(1.1) = 4 + .1\sqrt{2} \sim 4.141$
C. $L(1.1)$ is less than the true value of $f(1.1)$. The graph of $f'(x)$ is increasing for $x > 0$. Thus, the graph of f is concave up for $x > 0$. The tangent line to $f(x)$ at $x = 1$ would lie below the graph. Using $L(x)$ to approximate $f(x)$ for x near 1 will give underestimates.

7. A. 3 B. -5, 3, 5 C. 2
D. -5, 5 $g''(x)$ changes sign at -5 and 5, but not at 3

8. f'' changes sing at $x = ½$ from negative to positive. $f(x)$ is concave down for $x < ½$.

9. A. $x = -1, 2, 5$
B. g has a local max at $x = -1$, g has a local min at $x = 5$, g has neither at $x = 2$.
C. g is increasing on $(-5, -1)$ and $(5, 7)$.
D. g is decreasing on $(-1, 2)$ and $(2, 7)$.
E. g is concave up on $(-1/4, 2)$ and $(4, 7)$.
F. g is concave down on $(-5, -1/4)$ and $(2, 4)$.
G. 3, at $x = -1/4, 2, 4$

10. f ' changes sign from – to + at $x = 3$, so f changes from decreasing to increasing at $x = 3$.
f'' changes sign from – to + at $x = 1$, so f changes from concave down to concave up at $x = 1$.

11. f'' has 3 zeros, 0 and 3 multiplicity 2. f'' changes sign at $x = 0$, but not at $x = 3$, so f has only ONE inflection point at $x = 0$.

12. A. $x = -1, 3$ B. none, the derivative never changes sign
C. $(-\infty, -1)$ and $(-1, 3)$ and $(3, \infty)$ D. no where
E. $(-1, 1)$ and $(3, \infty)$ F. $(-\infty, -1)$ and $(1, 3)$
G. three, $x = -1, 1, 3$

13. A. The velocity function, m/sec
B. At time $t = 5$ seconds, the object is moving forward at a speed of 12 m/sec or the object has a velocity of 12 m/sec.
C. Yes, at times $t = \sim 8.2$ and 11.5 seconds as the velocity is zero at those values of t.
D. On the intervals (2.1, 5) and (8.2, 10) and (11.5, 12)
E. On the intervals (0, 2.1) and (5, 8.2) and (10, 11.5)
F. $t = \sim 8.2$, 11.5 seconds
G. Yes, at times $t = 5$, 10 seconds
H. Yes, on the interval (0, 2.1)

14. A. $x = 0, 23$
B. g has a local maximum at $x = 23$ since $g'(x)$ changes sign from = to – there.
C. (0, 23) D. (23,35)
E. (0, 5), (23, 25), (30, 35) F. (10, 15) and (20, 23)
G. $t = 35$ seconds H. $t = 0$, 23 seconds (at rest)

15. A. $x =-1, 3$ B. $(3, \infty)$ C. $(-\infty, -1)$ $(-1, 3)$
D. no where E. $x = 3$ F. $x = -1, 3/5$
G. $(-\infty, -1)$ $(3/5, \infty)$ H. $(-1, 3/5)$ I. $x = -1, 3/5$

16. A. $x = -5, 2$ B. $(-5, \infty)$ C. $(-\infty, -5)$
D. none E. $x = -5$ F. $x = 2, -8/3$
G. $(-\infty, -8/3)$, $(2, \infty)$ H. $x = -8/3, 2$

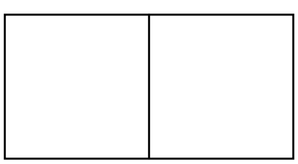
17. A. The top and bottom of the container are squares of dimension x by x. The four sides are rectangles of dimension x by y. The total surface area of the container is $2x^2 + 4xy$. The surface area is limited to 1000 square feet. Thus, $2x^2 + 4xy = 1000$. Solving this equation for y gives $y = (1000 – 2x^2)/4x$. The Volume of the container is $V = x^2y = x^2(1000 – 2x^2)/4x = -2x^3/4 + 250x$.

$V' = -6x^2/4 + 250$. $V' = 0$, $-6x^2 = -1000$ ➔ $x^2 = 1000/6$ ➔ $x \sqrt{\frac{1000}{6}} = \frac{10\sqrt{15}}{3} \sim 12.9$

$V'' = -12x/4 = -3x$ which is always ≤ 0. Thus, V is concave down everywhere and $x = \frac{10\sqrt{15}}{3} \sim 12.9$ is an absolute maximum. If $x = \frac{10\sqrt{15}}{3}$, then $y = \frac{10\sqrt{15}}{3}$.

The dimensions of the container should be $\frac{10\sqrt{15}}{3}$ ft. by $\frac{10\sqrt{15}}{3}$ ft. by $\frac{10\sqrt{15}}{3}$ ft.

B. The corral is sketched below. Let x be the vertical dimension and y be the horizontal dimension.



The interior area of the corral is $xy = 1300$ (the required area).
Solving for y: $y = 1300/x$.
The cost of the corral is Cost $= 5(2x + 2y) + 3x = 10(x + 1300/x) + 3x = 13x + 13000/x$.

$C = 13x + 13000/x$ ➔ $C' = 13 – 13000/x^2$; $C' = 0$ ➔ $-13000/x^2 = -13$ ➔ $13000 = 13x^2$ ➔ $x^2 = 1000$➔ $x = 10\sqrt{10}$. $C'' = 26000/x^3$ which is positive for $x \neq 0$. Thus, C is concave up everywhere, so $x = 10\sqrt{10}$ is and absolute minimum.

When $x = 10\sqrt{10}$, $y = 130/\sqrt{10} = 13\sqrt{10}$. The outside dimensions of the corral are $10\sqrt{10}$ yards by $13\sqrt{10}$ yards