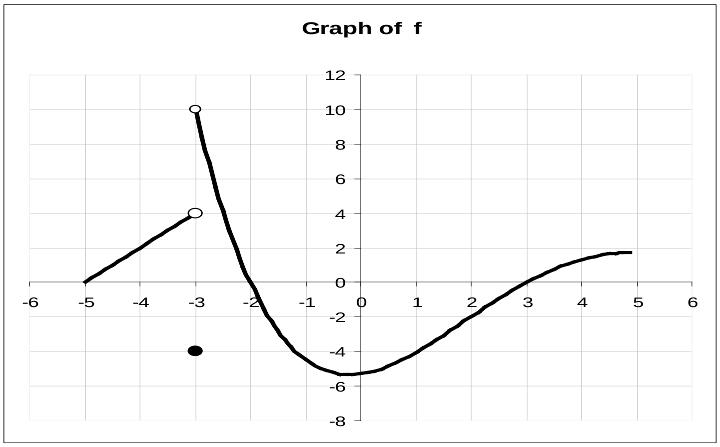
MA 181 BRUNETT REVIEW FOR FINAL EXAM

1. Use the graph of f below to answer the following questions:



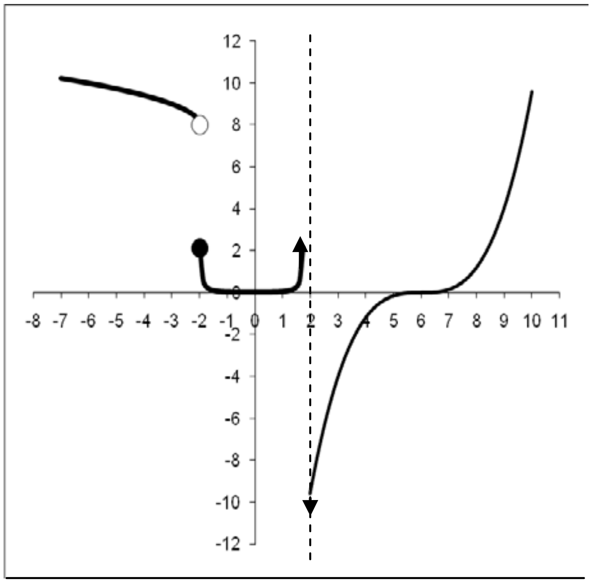
A. $\lim_{x \to -3^+} f(x)$ B. $\lim_{x \to -3^-} f(x)$ C. $\lim_{x \to -3} f(x)$

D. f(-3) E. $\lim_{x \to 2} f(x)$ F. f(2)

*G. Is $\lim_{h \to 0} \dfrac{f(2 + h) - f(2)}{h}$ positive, negative, or zero?

*H. Is $\lim_{x \to -1} \dfrac{f(x) - f(-1)}{x + 1}$ positive, negative, or zero?

2. Use the graph of f below to answer the following questions:



A. $\lim_{x \to -2^+} f(x)$ B. $\lim_{x \to -2^-} f(x)$ C. $\lim_{x \to -2} f(x)$ D. f(-2)

E. $\lim_{x \to 2^-} f(x)$ F. $\lim_{x \to 2^+} f(x)$ G. $\lim_{x \to 2} f(x)$

*H. Is $\lim_{h \to 0} \dfrac{f(4 + h) - f(4)}{h}$ positive, negative, or zero?

*I. Is $\lim_{x \to -5} \dfrac{f(x) - f(-5)}{x + 5}$ positive, negative, or zero?

3. Find the following limits. You must be able to support your answer with an explanation or show algebraic steps.

A. $\lim_{x \to 1} (17x^6 - x + 8)$ B. $\lim_{x \to 81} \frac{x - 81}{\sqrt{x} - 9}$ C. $\lim_{x \to \infty} \frac{9 + 5x}{15 - 2x}$

D. $\lim_{x \to \infty} \sqrt{\frac{x^2 + 9x^3}{2x^3 - 3x}}$ E. $\lim_{x \to -\infty} \frac{x^8 - 8}{x^4 + 9}$

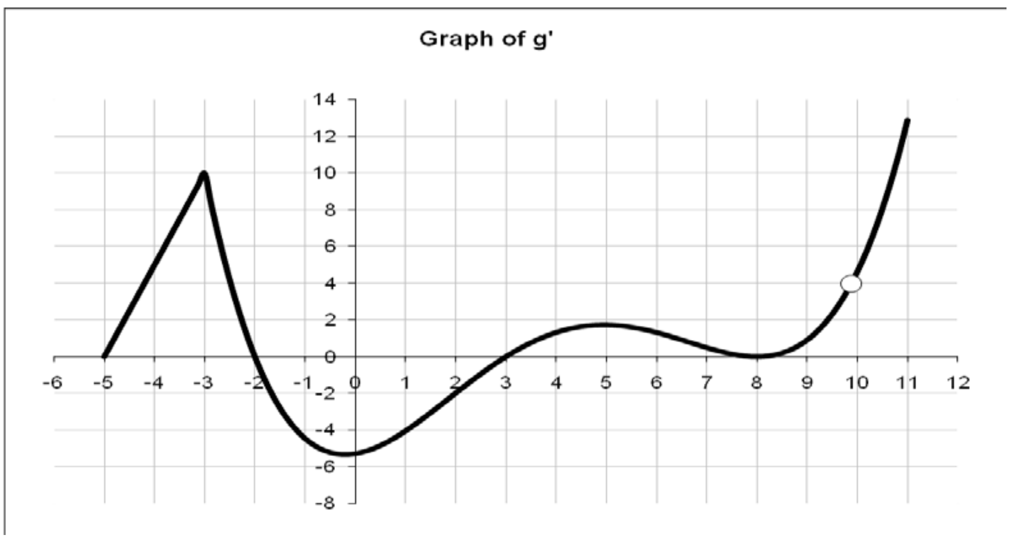
4. A. Find the value x at which the curve $y = \frac{x^2 - 25}{x^2 + 12x + 35}$ has a vertical asymptote.

B. Find the value y at which the curve $y = \frac{5x^3 - 16}{7x^3 - 5x + 4}$ has a horizontal asymptote.

5. Let f be a function with f(4) = 6 and whose derivative is $f'(x) = -(x - 2)^3$.
A. Write a linear approximation for f at x = 4.
B. Use the linear approximation to estimate f(4.2).
C. Is the approximation for f(4.2) greater than or less than the true value of f(4.2)? Explain.

6. The line tangent to the graph of y = f(x) at x = 3 passes through the points (-2, 3) and (4, -1).
A. Find f '(3).
B. Find the equation of the line tangent to f(x) at x = 3.
C. Find f(3).

7. Given the graph of y = g'(x), assume that g is defined for $-5 \le x \le 11$, answer the following:

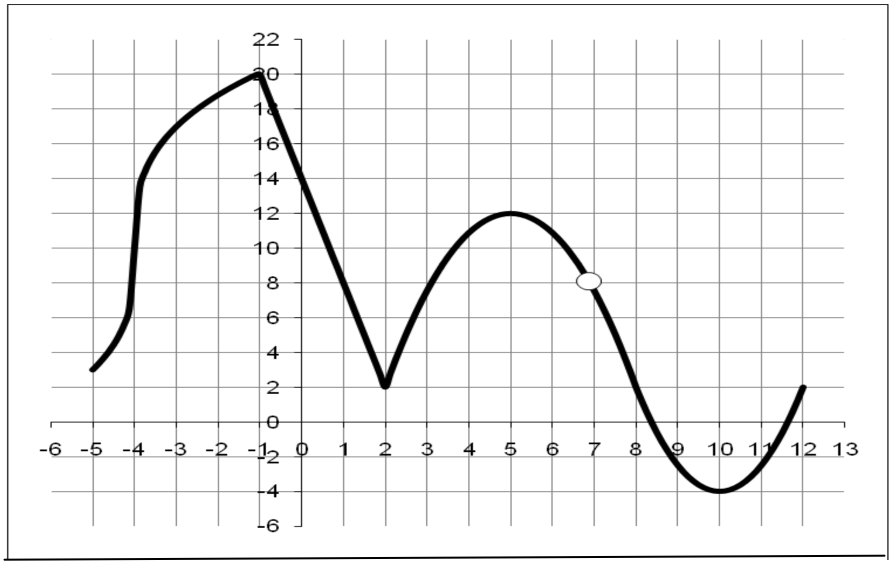


A. Where does g have critical numbers
B. Where does g have hypercritical numbers
C. Where is g increasing
D. Where is g decreasing
E. Where does g have a local maximum
F. Where does g have a local minimum
G. Where is g'(x) is increasing
H. Where is g'(x) is decreasing
I. Where is g"(x) > 0
J. Where is g"(x) < 0
K. Where does g have an inflection point

*L. Is $\lim_{h \to 0} \frac{g(2 + h) - g(2)}{h}$ positive, negative, or zero?

*M. Is $\lim_{x \to 7} \frac{g(x) - g(7)}{x - 7}$ positive, negative, or zero?

8. Given the graph of $y = g'(x)$, assume that g is defined for $-5 \le x \le 12$, answer the following:



A. Where does g have critical numbers

B. Where does g have hypercritical numbers

C. Where is g increasing

D. Where is g decreasing

E. Where does g have a local maximum

F. Where does g have a local minimum

G. Where is g'(x) is increasing

H. Where is g'(x) is decreasing

I. Where is g"(x) > 0

J. Where is g"(x) < 0

K. Where does g have an inflection point

*L. Is $\lim\limits_{h \to 0} \dfrac{g(5 + h) - g(5)}{h}$ positive, negative, or zero?

*M. Is $\lim\limits_{x \to 8} \dfrac{g(x) - g(8)}{x - 8}$ positive, negative, or zero?

9. Find the derivative, using the rules of differentiation, for each of the following functions:

A. $y = 7x^3 e^{x^5}$

B. $y = \dfrac{e^x}{x^4 + \sin x}$

C. $h(x) = 4x^5 - \dfrac{3}{x^3} + 2\sqrt{x}$

D. $y = \dfrac{3x^2}{\sqrt{x} + 4}$

E. Find $f^{(355)}(x)$ for $f(x) = e^x$

F. Find $f^{(886)}$ for $f(x) = \cos(3x)$

G. $y = \cos^5(x^3)$

H. $g(x) = 2xe^x + x\sin(3x)$

10. Find dy/dx using implicit differentiation.

A. $\tan(xy) = \sec x + y^2$

B. $x^2 e^{xy} = x \cos y$

11. Find the derivative

A. $y = (\sin x)^{x^2}$

B. $y = \ln\sqrt{\dfrac{3x^2 + 5}{x^3 + 4}}$

C. $y = \pi^e + \pi^x + x^\pi + (\pi^e)^x + x^{\pi^e}$

D. $y = \ln\dfrac{(3x^2 + 5)^3}{\sqrt{x^3 + 4}}$

E. $y = (2x^3 + 5)e^{x^2}$

*12. Suppose that the height of a projectile fired vertically upward from a height of 64 feet with an initial velocity of 48 feet per second is given by $h(t) = -16t^2 + 48t + 64$

A. What is the average velocity of the projectile for each of the following time intervals?

i. [3, 3.5] ii. [3, 3.1]

B. What is the instantaneous velocity of the projectile at t = 3 seconds?

C. How does the instantaneous velocity at t = 3 s related to the average velocities for the intervals in part A?

13. Use the properties of integrals to solve the following problems:

A. Simplify: $\int_2^{10} f(x)dx - \int_2^7 f(x)dx$

B. If $\int_0^1 f(t)dt = 2, \quad \int_0^4 f(t)dt = -6, \text{ and } \int_3^4 f(t)dt = 1, \text{ then find } \int_1^3 f(t)dt$

*14. An animal population is increasing at a rate of 200 + 50 t per year (where t is measured in years).

A. What integral would you set up to compute how much the animal population increases between the sixth and ninth year?

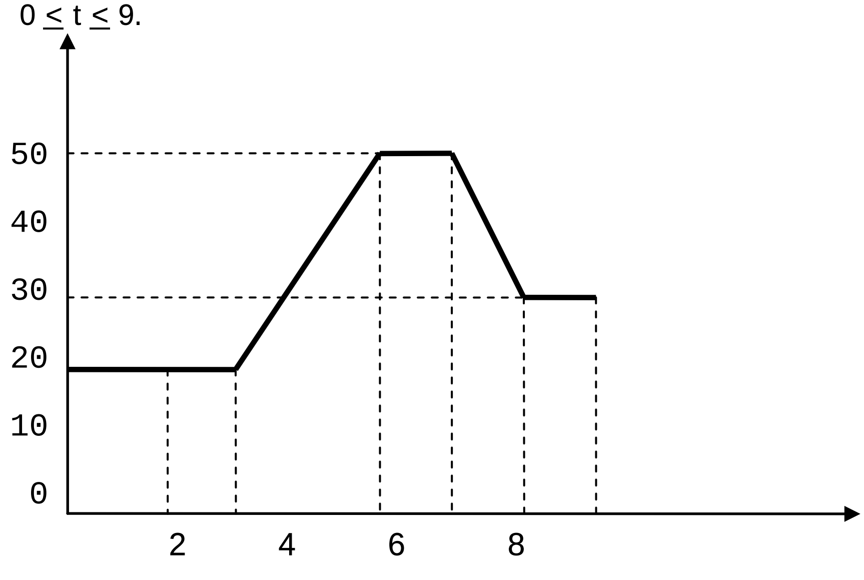
B. Compute the answer.

*15. Let W(t) be the weight of a child (in pounds) who is t years old. Interpret the following in practical terms.

A. $W(4) = 42$ B. $W'(2) = 10$

C. $\int_4^6 W'(t)dt = 20$ D. How much does the child weigh at 6 years old?

16. A dry ice puck is pushed on an uneven surface. Below is graph of the velocity v (in cm/s) as a function of the time t (in seconds). Determine the total distance traveled by the puck for $0 \le t \le 9$.



17. If $\int_0^3 f(x)\,dx = 12$, $\int_0^6 f(x)\,dx = 42$, and $\int_0^3 g(x)\,dx = 2$, find the value of the following:

A. $\int_3^6 f(x)\,dx$ B. $\int_3^6 \left(2f(x) - 3\right)dx$

C. $\int_0^3 \left[g(x) - 4f(x)\right]$ D. $\int_0^6 g(x)\,dx$

18. Evaluate the following integrals:

A. $\int_2^3 (2x - 6x^2)\,dx$

B. $\int_1^3 5^x\,dx$

C. $\int_1^3 \frac{x^2 - 4x}{x^2}\,dx$

D. $\int_1^7 \frac{1}{\sqrt[3]{t}}\,dt$

E. $\int x\left(5 + \sqrt{x}\right)\,dx$

F. $\int_0^{\frac{\pi}{6}} (1 + \cos\theta)\,d\theta$

19. If $\int_3^b 3x^2 dx = 37$, find the value of b.

20. A particle travels along a line. Its velocity in meters per second is given by $3t^2 - 12t$, $t > 0$. Find

A. the displacement from t = 0 to t = 8.

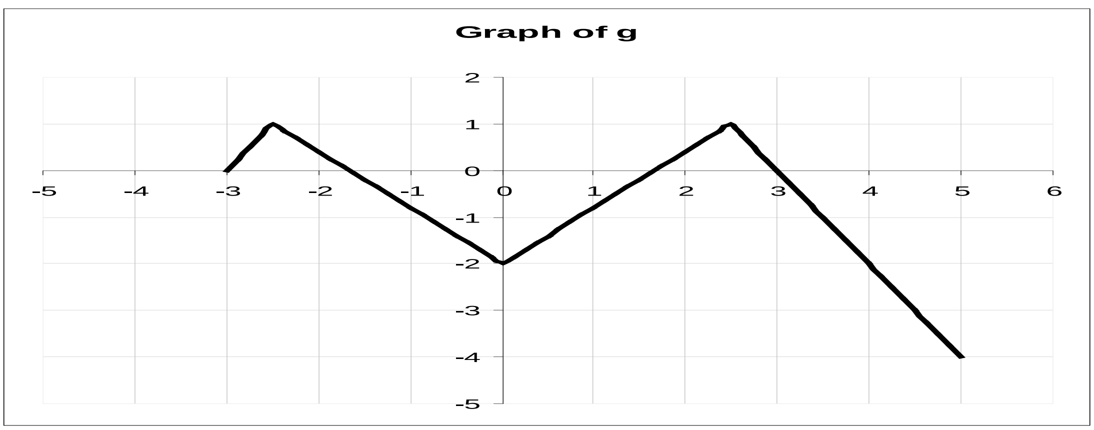
B. the distance traveled by the particle from t = 0 to t = 8.

C. when is the particle slowing down?

D. On the interval from (0, 4) when is the particle traveling the fastest?

*21. A. Let $f(x) = \frac{1}{2}\int_0^x t^3 dt$. Find the value of $f'(2)$. B. If $F(x) = \int_2^{3x} e^{t^4} dt$, find the value of $F'(x)$.

*22. Let $f(x) = \int_{-3}^x g(t)dt$ where g is the function whose graph is shown:



A. Evaluate: i. f(-3) ii. f(-1.5) iii. f(0) iv. f(1.5) v. f (3) vi. f(5)

B. On what interval(s) is f increasing?

C. On what interval(s) is f concave down?

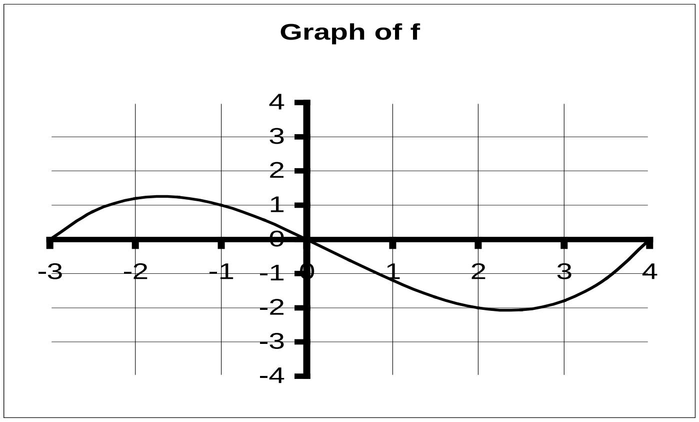
D. At what value(s) of x does f have an absolute maximum?

E. At what value(s) of x does f have an absolute minimum?

F. At what value(s) of x does f have inflection points?

G. Sketch a graph of f on [-3, 5].

*23. Let $g(x) = \int_{-3}^{x} f(t)\,dt$, where f is the function whose graph is shown. Assume g(-3) = 0.



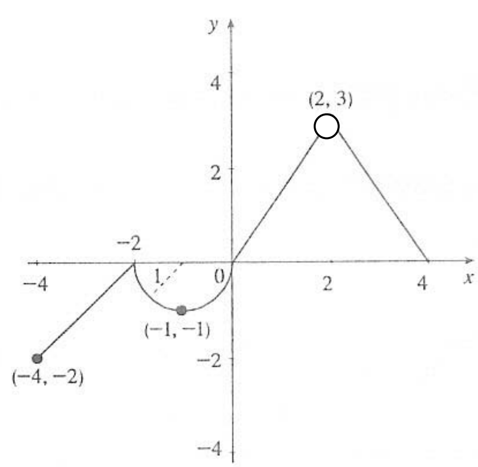
A. Approximate each of the following: i. g(-2) ii. g(0) iii. g(2) iv. g(4)
B. On what interval(s) is g increasing?
C. On what interval(s) is g concave down?
D. At what value(s) of x does g have an absolute maximum?
E. At what value(s) of x does g have an absolute minimum?
F. At what value(s) of x does g have inflection points? G. Sketch a graph of g on [-3, 4].

*24. Optimization Problems

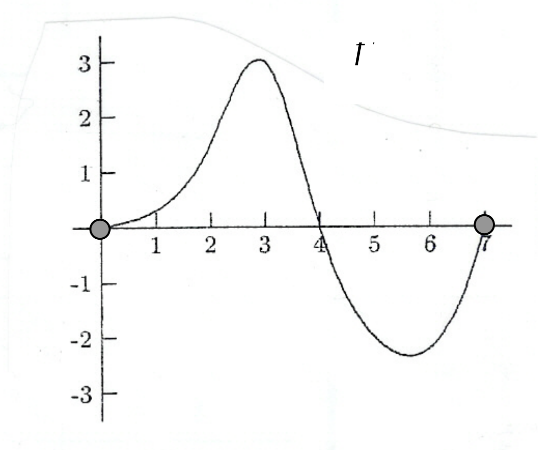
A. A farmer has 20 feet of fence, and he wants to make from it a rectangular pen for his pig, using a barn as one of the sides. In square feet, what is the maximum area possible for this pen?
B. A farmer wants to fence in a rectangular plot in a large field. A rock wall that already exists will be used for the north boundary. The fencing for the east and west sides of the plot will cost $3 per yard. The farmer has to use special fencing for the south side that cost $5 per yard. If the area of the plot must be 600 square yards,
   i. Find the dimensions that will minimize the cost.
   ii. What is the minimum cost?

25. Use the graph of *f* at the right to answer the following questions.

A. What are the domain and range
B. Where is *f* discontinuous?
C. Find $\lim_{x\to 2} f(x)$.
D. Find $\lim_{x\to -4^+} f(x)$.
E. Find $f'(-3)$.
F. Find $f'(-1)$.
G. Given $F(x) = \int_{-2}^{x} f(t)\,dt$,
   i. Evaluate F(0)
   ii. Evaluate F'(-1)
   iii. Over what intervals shown on the graph is F concave down? Explain/justify your answer.

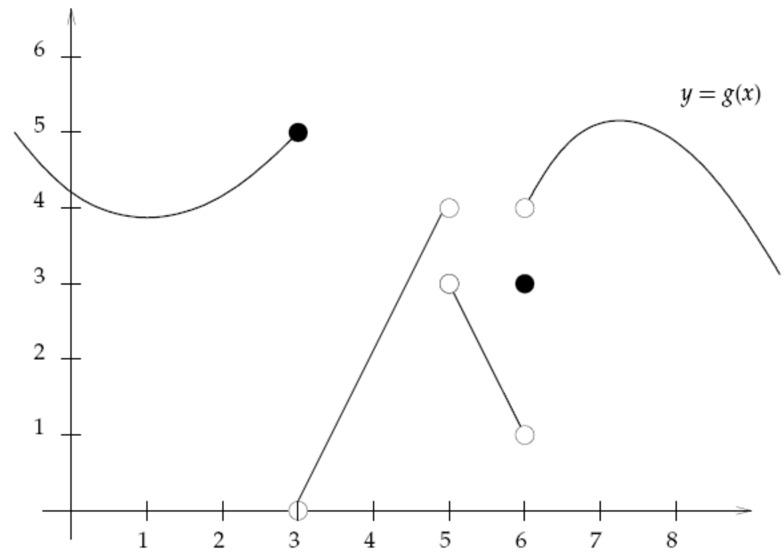


26. The graph below represents the derivative of the function $f(x)$; that is, the graph is $f'(x)$.



A. On which intervals is f(x) decreasing?
B. On which intervals is f(x) concave down?
C. Does f have any local extrema?
If there are not any write *none*.
D. local minima: _________
E. local maxima: _________
F. The point $(3, f(3))$ is a/an $\begin{cases} \text{zero of } f \\ \text{local maximum of } f \\ \text{inflection point of } f \end{cases}$

27. Use the graph of the function *g* to find the following.



a. $\lim_{x \to 6^-} g(x) =$

b. $\lim_{h \to 0} \frac{g(5.5+h) - g(5.5)}{h} =$

c. $\lim_{x \to 4^+} \frac{x-5}{g(x)}$

d. State the intervals where g is continuous.

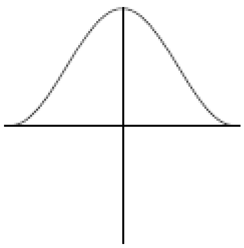
e. The function g has a(n) $\begin{cases} \text{infinite} \\ \text{jump} \\ \text{removable} \end{cases}$ discontinuity at x = 6. (*circle one)*

f. $g''(8.5)$ is $\begin{cases} \text{negative} \\ \text{zero} \\ \text{positive} \end{cases}$.

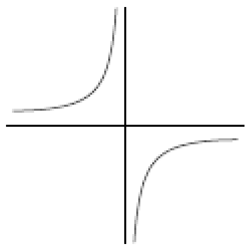
28. For each of the graphs *a – d*, determine which of the graphs *I – V* is the graph of its derivative. Write your answer in the space provided.

Original Functions

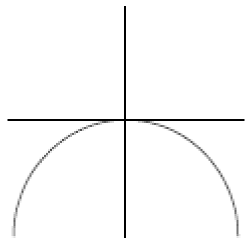
a. ____



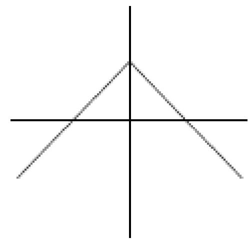
b. ____



c. ____

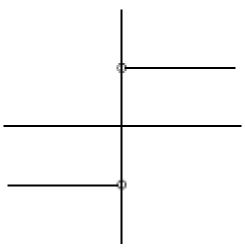


d. ____

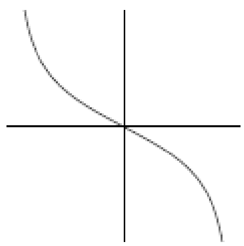


Derivatives

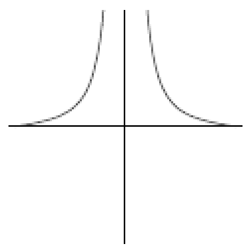
I.



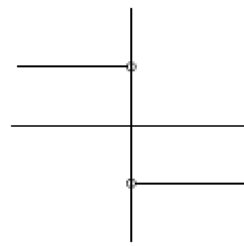
II.



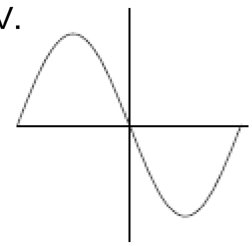
III.



IV.



V.



ANSWERS:

1. A. 10 B. 4 C. dne D. -4 E. -2 F. -2
   G. positive H. negative

2. A. 2 B. 8 C. dne D. 2 E. $+\infty$ or dne F. $-\infty$ or dne
   G. dne H. positive I. negative

3. A. 24 by direct sub. this function is continuous B. 18 (by factoring)
   C. -5/2, this function has a H.A. of y = -5/2
   D. $\frac{3}{\sqrt{2}}$, Applied Limit Law and noted that the quotient function has a H.A. of y = 9/2.
   E. ∞, this function has an oblique asymptote

4. A. x = -7 B. y = 5/7

5. A. y = 6 – 8(x – 4) B. 4.4
   C. Overestimate, f’ is decreasing at x = 4, so f” < 0, and f is concave down, the tangent line lies above f.

6. A. $-\frac{2}{3}$ B. $y = -\frac{2}{3}x + \frac{5}{3}$ or $y = -\frac{2}{3}(x-3) - \frac{1}{3}$ C. $-\frac{1}{3}$

7.
A. x = -5, -2, 3, 8, and 10
B. x = -3, 0, 5, 8, and (10)
C. (-5, -2), (3, 8) and (8, 10), (10, 11)
D. (-2, 3)
E. x = -2
F. x = 3
G. (-5, -3), (0, 5), and (8, 11)
H. (-3, 0) and (5, 8)
I. same as G
J. same as H
K. x = -3, 0, 5, 8
L. negative, read the graph g'(2) < 0
M. positive, read the graph g'(7) > 0

8.
A. x = 7, 8.5, 11.5
B. x = -4, -1, 2, 5, (7), and 10
C. (-5, 7), (7, 8.5) and (11.5, 12)
D. (8.5, 11.5)
E. x = 8.5
F. x = 11.5
G. (-5, -1), (2, 5), and (10, 12)
H. (-1, 2), (5, 7), (7, 10)
I. same as G
J. same as H
K. x = -1, 2, 5, , 10
L. positive
M. positive

9.
A. $y' = 21x^2 e^{x^5} + 7x^3 \bullet 5x^4 e^{x^5} = 7x^2 e^{x^5}\left(3 + 5x^5\right)$

B. $y' = \dfrac{e^x\left(x^4 + \sin x\right) - e^x\left(4x^3 + \cos x\right)}{\left(x^4 + \sin x\right)^2} = \dfrac{e^x\left(x^4 - 4x^3 + \sin x - \cos x\right)}{\left(x^4 + \sin x\right)^2}$

C. $h'(x) = 20x^4 + \dfrac{9}{x^4} + \dfrac{1}{\sqrt{x}}$

D.

$$y' = \frac{6x\left(\sqrt{x} + 4\right) - \left(3x^2\right)\frac{1}{2\sqrt{x}}}{\left(\sqrt{x} + 4\right)^2} = \frac{\frac{12x^2 + 48x\sqrt{x} - 3x^2}{2\sqrt{x}}}{\left(\sqrt{x} + 4\right)^2} = \frac{12x^2 + 48x\sqrt{x} - 3x^2}{2\sqrt{x}\left(\sqrt{x} + 4\right)^2}$$

E. $f^{(355)}(x) = e^x$
F. $f^{(886)}(x) = -3^{886}\cos(3x)$

G. $y' = -5\cos^4(x^3)\sin(x^3)(3x^2)$
H. $g'(x) = 2e^x(1 + x) + \sin(3x) + 3x\cos(3x)$

10.
E. $y' = \dfrac{\sec x \tan x - y\sec^2(xy)}{x\sec^2(xy) - 2y}$
F. $y' = \dfrac{\cos y - 2xe^{xy} - x^2ye^{xy}}{x^3 e^{xy} + x\sin y}$

11.
A. $y' = \sin x^{x^2}\left[2x\ln\sin x + x^2\cot x\right]$

B. $y' = \dfrac{1}{2}\left[\dfrac{6x}{3x^2 + 5} - \dfrac{3x^2}{x^3 + 4}\right]$

C. $y' = 0 + (\ln\pi)\pi^x + \pi x^{\pi - 1} + \ln\left(\pi^e\right)\left(\pi^e\right)^x + \pi^e x^{\pi^e - 1}$

D. $y = 3\ln\left(3x^2 + 5\right) - \dfrac{1}{2}\ln\left(x^3 + 4\right)$ and $y' = 3\left(\dfrac{6x}{3x^2 + 5}\right) - \dfrac{1}{2}\left(\dfrac{3x^2}{x^3 + 4}\right)$

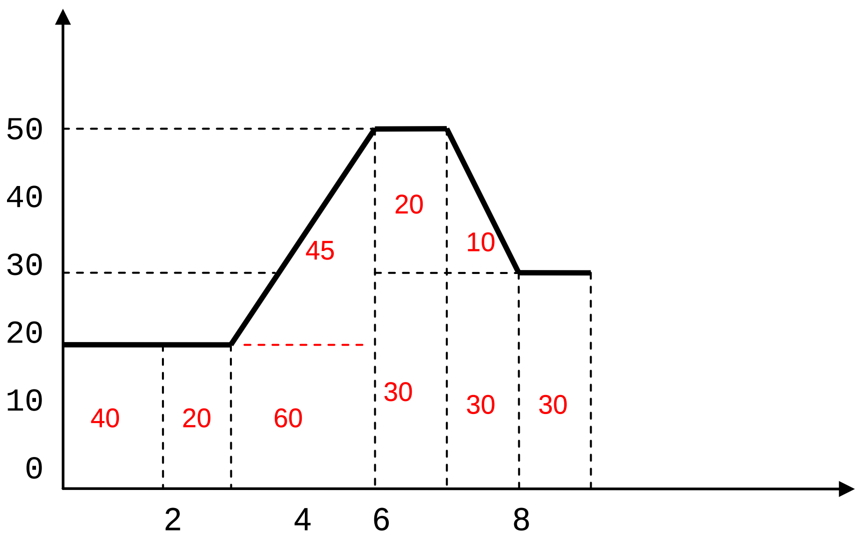
E. $y' = \left(6x^2\right)e^{x^2} + \left(2x^3 + 5\right)2xe^{x^2} = 2xe^{x^2}\left(2x^3 + 3x + 5\right)$

12.
A. i. -56 ft/sec ii. -49.6 ft/sec
B. h'(3) = -32(3) + 48 = -48 ft/sec
C. The instantaneous velocity is a limit of the average velocities as t approaches 3.

13. A. $\int_7^{10} f(x)dx$ B. -9 14. A. $\int_6^9 (200 + 50t)\,dt$ B. 1725 animals

15. A. At age 4 the child weighs 42 pounds.
B. At age 2 the child is gaining weight at the rate of 10 lbs./year
C. The child gains 20 pounds from age 4 to age 6.
D. 42 (weight at age 4) + 20 (amount gained from age 4 to 6) = 62 lbs.

16. (40 + 20 + 60 + 45 + 20 + 10 + 3(30)) = 285 cm



17. A. 30 B. 51 C. -46 D. Can't be determined

18. A. -33 B. 120/ln(5) C. -4 ln 3 + 2 D. $\frac{3\sqrt[3]{49}}{2} - \frac{3}{2}$

E. $\frac{2}{5}x^{5/2} + \frac{5}{2}x^2 + C$ F. $\frac{1}{2} + \frac{\pi}{6}$

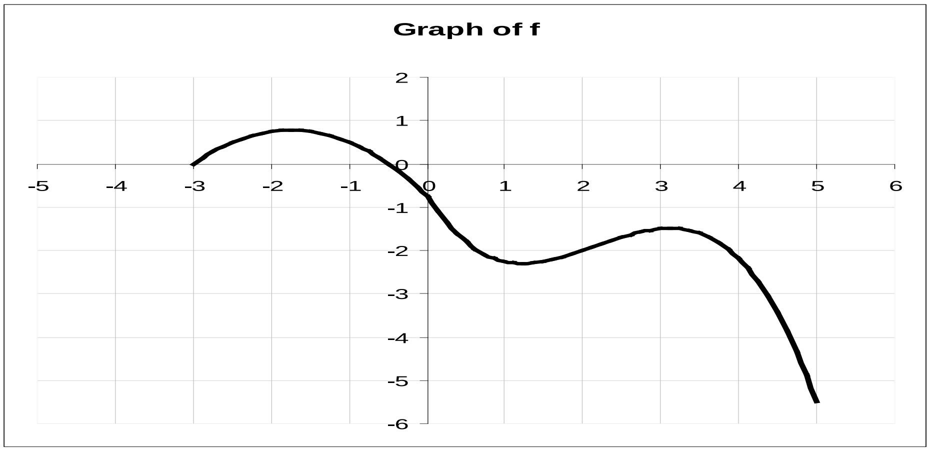
19. 4

20. A. 128 m B. 192 m

21. A. 4 B. $3e^{81x^4}$ or $3e^{(3x)^4}$

22. A. i. 0
ii. 0.75 (triangle base = 1.5 and height = 1)
iii. -0.75 (0.75 plus the negative area from triangle with base = 1.5 and height = 2)
iv. -2.25 (-0.75 plus the negative area from triangle with base = 1.5 and height = 2)
v. -1.5 (-2.25 plus the positive area from triangle with base = 1.5 and height = 1)
vi. -5.5 (-1.5 plus the negative area from triangle with base = 2 and height = 4)

B. (-3, -1.5) and (1.5, 3) {where the graph of g is above the x-axis}
C. (-2.5, 0) and (2.5, 5) {where the graph of g is decreasing}
D. x = -1.5
E. x = 5
F. x = -2.5, 0, 2.5 {where the graph of g changes direction}
G. Note: The graph does not portray the inflection point at x = -2.5.



23. A. i. 0.5 ii. 2 iii. 0 iv. -3

B. (-3, 0)

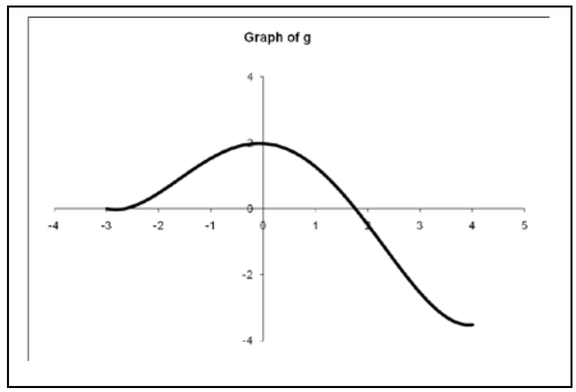
C. (-1.5, 2.5)

D. x = 0

E. x = 4

F. x = -1.5, 2.5

G.



24. A. The area A = xy and the perimeter P = 2y + x = 20. Solving for y, gives y = 10 – x/2. $A = x(10 - x/2) = 10x - x^2/2$. A' = 10 – x. A' = 0 gives x = 10. Since A" = -1 < 0 everywhere, A is concave down everywhere and x = 10 is an absolute maximum for A. The dimensions are x = 10 feet and y = 10 – 10/2 = 5 feet.

B. Area = xy = 600, solving for y gives y = 600/x.
The cost C = 2(3y) + 5x = 6y + 5x = 6(600/x) + 5x = 3600/x + 5x
$C' = -3600/x^2 + 5 = 0 \Rightarrow 5x^2 = 3600 \Rightarrow x^2 = 720 \Rightarrow x = 12\sqrt{5}$
$x = 12\sqrt{5}$ gives an absolute minimum value for C.
Verify that C" > 0, for x > 0 so C is concave up for x > 0 and has an absolute minimum value for x > 0.
$y = 600/(12\sqrt{5}) = 50/\sqrt{5} = 10\sqrt{5}$

i. The dimensions are $12\sqrt{5}$ feet by $10\sqrt{5}$ feet.

ii. The minimum cost is $6(10\sqrt{5}) + 5(12\sqrt{5})$ ~ $68.31

25. A. domain: [-4, 2) and (2, 4] range: [-2, 3)

B. x = 2 C. 3 D. -2

E. 1 F. 0

G. i. – pi/2, ii. -1 iii. (-2, -1) and (2, 4), F' = f is decreasing.

26. A. (4, 7) B. (3, 6) C. yes D. none E. x = 4 F. inflection point of f

27. A. 1 B. -2 C. -1/2 D. (-1, 3), (3, 5), (5, 6), (6, 9)

E. jump F. negative

28. A. ii B. iii C. v D. iv