1. The difference quotient (i.e. slope) between two points (a, f(a)) and (a + h, f(a + h)) on a curve f(x) is $\dfrac{f(a+h)-f(a)}{(a+h)-a} = \dfrac{f(a+h)-f(a)}{h}$.

For $f(x) = 4x^2 - 3x + 7$, simplify the difference quotient for a = 2.

$$\frac{f(a+h)-f(a)}{h} = \frac{f(2+h)-f(2)}{h}$$

$$= \frac{\left[4(2+h)^2 - 3(2+h) + 7\right] - \left[4(2)^2 - 3(2) + 7\right]}{h}$$

$$= \frac{\left[4(4+4h+h^2) - (6+3h) + 7\right] - \left[4(4) - 6 + 7\right]}{h}$$

$$= \frac{\left[16 + 16h + 4h^2 - 6 - 3h + 7\right] - \left[16 - 6 + 7\right]}{h}$$

$$= \frac{\left[10 + 13h + 4h^2 + 7\right] - \left[10 + 7\right]}{h} = \frac{\left[17 + 13h + 4h^2\right] - 17}{h}$$

$$= \frac{13h + 4h^2}{h} = \frac{h\,13 + 4h}{h} = 13 + 4h$$

2. Let $f(x) = \dfrac{5x^2 - 3x - 14}{x^2 - 4}$, what is the value of f as x approaches 2?

f(2) is undefined. However, $\dfrac{5x^2 - 3x - 14}{x^2 - 4} = \dfrac{5x+7\quad x-2}{x-2\quad x+2} = \dfrac{5x+7}{x+2}$

which gives the value of f(x) everywhere except at x = 2.

$\dfrac{5x+7}{x+2}$ = [5(2) + 7]/[2 + 2] = 17/4, for x = 2. Thus, f(x) must be close to 17/4 when x is close to 2.

3. Let $g(x) = \dfrac{x-1}{\sqrt{x}-1}$, what is the value of g as x approaches 1?

g(1) is undefined. However, $\dfrac{x-1}{\sqrt{x}-1} = \dfrac{\sqrt{x}-1\quad \sqrt{x}+1}{\sqrt{x}-1} = \sqrt{x} + 1$, which gives the value of g(x) everywhere except at x = 1. $\sqrt{x} + 1$ is 2 for x = 1. Thus, g(x) must be close to 2 when x is close to 1.

4. Simplify: $h(x) = \dfrac{x^8 - 3\sqrt{x} + 2}{\sqrt{x}}$

$h(x) = x^{\frac{15}{2}} - 3 + 2x^{-\frac{1}{2}}$

5. Write in the form x^n where n is a rational number: $\sqrt[7]{x^4}$

$x^{\frac{4}{7}}$

6. How many constant terms does k(x) have?

$k(x) = 3x^{\pi} + \pi + \sqrt{3} + e^2 + \pi x + e^2 x + e^2 x^3 + 5$

k has 4 constant terms: π, $\sqrt{3}$, e^2, 5

7. Given $m(x) = \begin{cases} x^2 - 4 & \text{if } x < 3 \\ 0 & \text{if } x = 3 \\ x + 2 & \text{if } x > 3 \end{cases}$

Find:

A. m(3)

B. m(-3)

C. m(5)

0

$(-3)^2 - 4 = 5$

5 = 2 = 7

8. Given $n(x) = \dfrac{3x^2 + 13x - 10}{x^2 - 25} = \dfrac{3x - 2 \quad x + 5}{x - 5 \quad x + 5}$

Find:

A. The vertical asymptote(s) of n.

$x = 5$

B. The horizontal asymptote(s) of n.

$y = 3$