MA091 Beginning Algebra Summary

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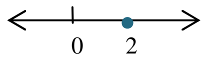
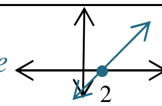
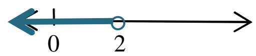
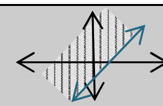
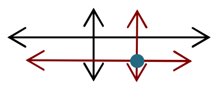
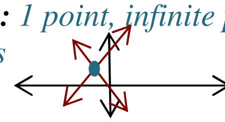
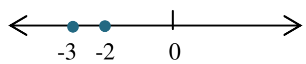
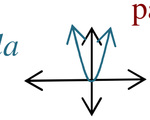
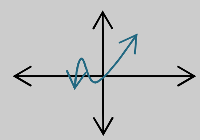
Big Picture – Algebra is Solving Equations with Variables*

	1 Variable	2 Variables
Linear Equations	$x - 2 = 0$ MA090 **Solution:** *1 Point* 	$y = x - 2$ page 5 **Solution:** *Line* 
Linear Inequalities	$x - 2 < 0$ page 2 **Solution:** *Ray* 	$y > x - 2$ **Solution:** *½ plane* 
Systems of Linear Equations	$\begin{cases} x = 7 \\ y = -5 \end{cases}$ page 7 **Solution:** *1 point, infinite points or no points* 	$\begin{cases} y = -x \\ y = x + 2 \end{cases}$ page 7 **Solution:** *1 point, infinite points or no points* 
Quadratic Equations	$x^2 + 5x + 6 = 0$ page 15 **Solution:** *Usually 2 points* 	$y = 2x^2$ page 16 **Solution:** *Parabola* 
Higher Degree Polynomial Equations (cubic, quartic, etc.)	$x^3 + 5x^2 + 6x = 0$ page 15 **Solution:** *Usually x points, where x is the highest exponent* 	$y = x^3 + 5x^2 + 6x$ **Solution:** *Curve* 
Rational Equations	$\frac{x^2 - 1}{x + 1} = 1$ page 20 **Solution:** *Usually simplifies to a linear or quadratic equation*	$y = -\frac{x^2 - 1}{x + 1} + 1$ **Solution:** *Usually simplifies to a linear or quadratic equation*

* To determine the equation type, simplify the equation. Occasionally all variables "cancel out".

- If the resulting equation is true (e.g. 5 = 5), then all real numbers are solutions.
- If the resulting equation is false (e.g. 5 = 4), then there are no solutions.

Find It Fast

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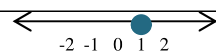
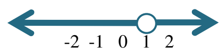
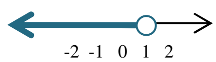
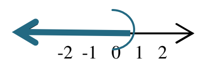
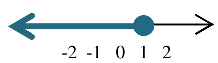
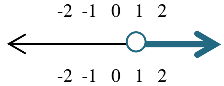
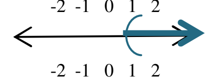
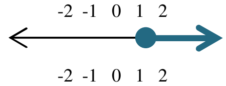
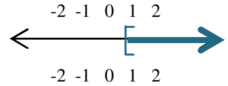
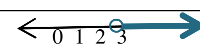
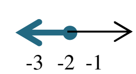
Number Lines & Interval Notation

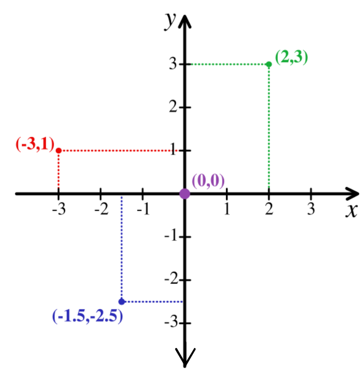
Number Lines	○ ( ) – If the point is not included ● [ ] – If the point is included —— – Shade areas where infinite points are included
Interval Notation (shortcut, instead of drawing a number line)	▪ **1st graph the answers on a number line**, then write the interval notation by following your shading from left to right ▪ Always written: 1) Left enclosure symbol, 2) **smallest number**, 3) **comma**, 4) **largest number**, 5) right enclosure symbol ▪ Enclosure symbols ( ) – Does not include the point [ ] – Includes the point ▪ Infinity can never be reached, so the enclosure symbol which surrounds it is an open parenthesis

Ex. $x=1$ "x is equal to 1"			$\{1\}$
Ex. $x\neq 1$ "x is not equal to 1"			
Ex. $x<1$ "x is less than 1"			$(-\infty,1)$
Ex. $x\leq 1$ "x is less than or equal to 1" . .			$(-\infty,1]$
Ex. $x>1$ "x is greater than 1"			$(1,\infty)$
Ex. $x\geq 1$ "x is greater than or equal to 1"			$[1,\infty)$

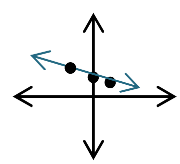
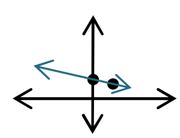
Linear Inequalities with 1 Variable

Standard Form	▪ $ax+b<c$ $ax+b\leq c$ $ax+b>c$ $ax+b\geq c$	▹ $2x+4>10$
Solution	▪ A ray	▹ $x>3$ 
Multiplication Property of Inequality	▪ When both sides of an inequality are multiplied or divided by a negative number, the direction of the inequality symbol must be reversed to form an equivalent inequality.	▹ $4\leq -2x$ $\frac{4}{-2}\geq\frac{-2x}{-2}$
Solving	1. Same as *Solving an Equation with 1 Variable (MA090)*, except when both sides are multiplied or divided by a negative number	Ex $4\leq -2x$ $\frac{4}{-2}\geq\frac{-2x}{-2}$ $-2\geq x$ $x\leq -2$ 
	2. Checking ▪ Plug solution(s) into the original equation. Should get a true inequality. ▪ Plug a number which is not a solution into the original equation. Shouldn't get a true inequality	▹ $4\leq -2(-3)$ $4\leq 6$ √ ▹ $4\leq -2(0)$ $4\leq 0$ ×

The Cartesian Plane

Rectangular Coordinate System	▪ Two number lines intersecting at the point 0 on each number line. ▪ X-AXIS - The horizontal number line ▪ Y-AXIS - The vertical number line ▪ ORIGIN - The point of intersection of the axes ▪ QUADRANTS - Four areas which the rectangular coordinate system is divided into ▪ ORDERED PAIR - A way of representing every point in the rectangular coordinate system (x,y)	Quadrant II Quadrant I  Quadrant III Quadrant IV
Is an Ordered Pair a Solution?	▪ Yes, if the equation is a true statement when the variables are replaced by the values of the ordered pair	Ex $x + 2y = 7$ $(1, 3)$ is a solution because $1 + 2(3) = 7$

Graphing Lines

General	▪ Lines which intersect the x-axis contain the variable x ▪ Lines which intersect the y-axis contain the variable y ▪ Lines which intersect both axis contain x and y	
Graphing by plotting random points	1. Solve equation for y 2. Pick three **easy** x-values & compute the corresponding y-values 3. Plot ordered pairs & draw a line through them. (If they don't line up, you made a mistake)	▷ $x + 2y = 7$ $y = -\frac{x}{2} + \frac{7}{2}$ x / y: -1 / 4; 0 / 3.5; 1 / 3 
Graphing linear equations by using a point and a slope	1. Plot the point 2. Starting at the plotted point, vertically move the rise of the slope and horizontally move the run of the slope. Plot the resulting point 3. Connect both points	▷ $y = -\frac{1}{2}x + \frac{7}{2}$ Point = 7/2 Slope = −1/2 

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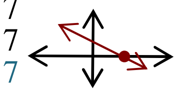
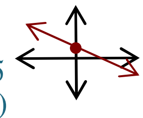
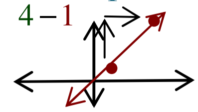
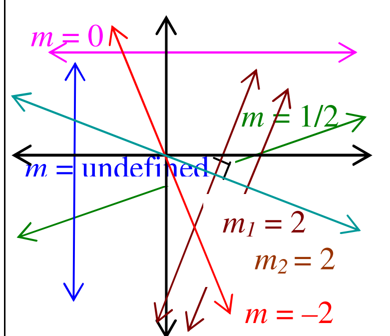
Line Basics

***x*-intercept** (x, 0)	▪ WHERE THE GRAPH CROSSES THE X-AXIS ▪ Let $y = 0$ and solve for x	Ex $x + 2y = 7$ $x + 2(0) = 7$ $x = 7$ $(7, 0)$ 
***y*-intercept** (0, y)	▪ WHERE THE GRAPH CROSSES THE Y-AXIS ▪ Let $x = 0$ and solve for y	Ex $x + 2y = 7$ $0 + 2y = 7$ $y = 3.5$ $(0, 3.5)$ 
Slope of a Line	▪ The slant of the line. Let Point 1: $P_1 = (x_1, y_1)$ & Point 2: $P_2 = (x_2, y_2)$ $m \text{ (slope)} = \dfrac{\text{rise (change in y)}}{\text{run (change in x)}} = \dfrac{y_2 - y_1}{x_2 - x_1}$	Ex Let $P_1 = (\overset{x_1}{1}, \overset{y_1}{1})$, $P_2 = (\overset{x_2}{4}, \overset{y_2}{4})$ $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4-1}{4-1} = 1$ 
Properties of Slope	▪ POSITIVE SLOPE - Line goes up (from left to right). The greater the number, the steeper the slope ▪ NEGATIVE SLOPE - Line goes down (from left to right). The smaller the number (more negative), the steeper the slope. ▪ HORIZONTAL LINE - Slope is 0 ▪ VERTICAL LINE - Slope is undefined ▪ PARALLEL LINES - Same slope ▪ PERPENDICULAR LINES - The slope of one is the negative reciprocal of the other Ex: $m = -½$ is perpendicular to $m = 2$	
Standard Form	▪ $ax + by = c$ ▪ x and y are on the same side ▪ The equations contains no fractions and a is positive	▹ $x + 2y = 7$
Slope-Intercept Form	▪ $y = mx + b$, where m is the slope of the line, & b is the y-intercept ▪ "y equals form"; "easy to graph form"	▹ By solving $x + 2y = 7$ for y $y = -\dfrac{x}{2} + \dfrac{7}{2}$
Point-Slope Form	▪ $y - y_1 = m(x - x_1)$, where m is the slope of the line & (x_1, y_1) is a point on the line ▪ Simplified, it can give you Standard Form or Slope-Intercept Form	▹ Using $(7, 0)$ and $m = -\dfrac{1}{2}$ $y - 0 = -\dfrac{1}{2}(x - 7)$

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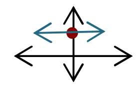
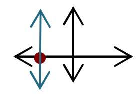
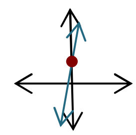
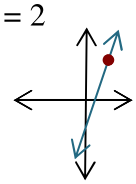
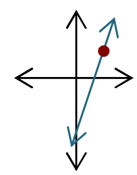
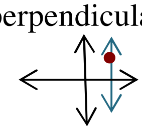
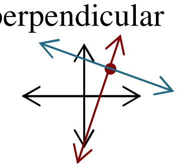
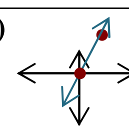
Finding the Equation of a Line

If you have a horizontal line…	▪ The slope is zero ▪ $y = b$, where b is the y-intercept	Ex. $y = 3$ 
If you have a vertical line…	▪ The slope is undefined ▪ $x = c$, where c is the x-intercept	Ex. $x = -3$ 
If you have a slope & y-intercept…	▪ Plug directly into *Slope-Intercept Form*	Ex. $m = 4$ **&** y-intercept $(0, 2)$ $y = 4x + 2$ $2 = 4(0) + 2$ √ 
If you have a point & a slope…	▪ METHOD 1 1. Use *Point-Slope Form* 2. Work equation into *Standard Form* or *Slope-Intercept Form*	Ex. point (3,2) & $m = 2$ $y - 2 = 2(x - 3)$ $y - 2 = 2x - 6$ $y = 2x - 4$ $(2) = 2(3) - 4$ √ 
	▪ METHOD 2 1. Plug the point into the *Slope-Intercept Form* and solve for b 2. Use values for m and b in the *Slope-Intercept Form*	Ex. point (3,2) & $m = 2$ $y = mx + b$ $(2) = (2)(3) + b$ $2 = 6 + b$ $b = -4$ $y = 2x - 4$ $(2) = 2(3) - 4$ √ 
If you have a point & a line that it is parallel or perpendicular to…	1. Determine the slope of the parallel or perpendicular line (e.g.. if it is parallel, it has the same slope) 2. If the slope is undefined or 0, draw a picture 3. If the slope is a non-zero real number, go to *If you have a point & a slope…*	Ex. point (3,2) & perpendicular to x-axis m = undefined $x = 3$  Ex. point (3,2) & perpendicular to $y = 2x - 4$ $m = 2$, so for perpendicular line $m = -1/2$ 
If you have 2 points…	1. Use the slope equation to determine the slope 2. Go to *If you have a point & a slope…*	Ex. $(0, 0)$ **&** $(3, 6)$ $m = \frac{6-0}{3-0} = 2$ 

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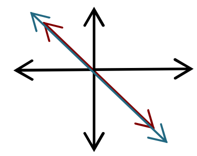
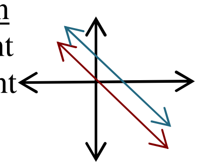
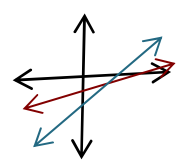
Systems of Linear Equations

Type of Intersection	▪ IDENTICAL (I) - Same slope & same y-intercept ▪ NO SOLUTION (N) - Same slope & different y-intercept, the lines are parallel ▪ ONE POINT - Different slopes	Solve $\begin{cases} y = -x \\ y = -2x \end{cases}$ $y = -x$ Identical Consistent Dependent 
Terminology	▪ CONSISTENT SYSTEM - The lines intersect at a point or are identical. System has at least 1 solution ▪ INCONSISTENT SYSTEM - The lines are parallel. System has no solution	Solve $\begin{cases} y = -x \\ y = -x + 1 \end{cases}$ No solution Inconsistent Independent 
	▪ DEPENDENT EQUATIONS - The lines are identical. Infinite solutions ▪ INDEPENDENT EQUATIONS - The lines are different. One solution or no solutions	
Solving by Graphing	1. Graph both equations on the same Cartesian plane. See *Graphing Lines p.3* 2. The intersection of the graphs gives the common solution(s). If the graphs intersect at a point, the solution is an ordered pair. 3. Check the solution in both original equations	Solve $\begin{cases} x - 2y = 1 \\ 2x - 2 = 2y \end{cases}$ $y = \frac{1}{2}x - \frac{1}{2}$ $y = x - 1$ One point Consistent Independent 

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Solving Systems of Linear Equations

Solving by Substitution	1. Use either equation to solve for 1 variable (pick easiest variable in easiest equation) 2. Substitute expression into the other equation 3. Solve the resulting 1 variable linear equation* 4. Substitute the value form Step 3 into either original equation to find the value of the other variable. 5. Solution is an ordered pair 6. Check the solution in both original equations	Solve $\begin{cases} x-2y=1 \\ 2x-4=6y \end{cases}$ 1. $y=\frac{1-x}{-2}$ 2. $2x-4=\overset{-3}{\cancel{6}}\left(\frac{1-x}{-\cancel{2}}\right)$ 3. $2x=-3+3x+4$ $x=-1$ 4. $y=\frac{1-(-1)}{-2}=-1$ 5. $(-1,-1)$ 6. $(-1)-2(-1)=1$ $1=1\surd$ $2(-1)-4=6(-1)$ $-6=-6\surd$
Solving by Addition or Subtraction of Equations	1. Rewrite each equation in standard form *Ax* + *By* = *C* 2. You want to be able to add the equations and have one variable cancel out. It is usually necessary to multiply one or both equations by a "magic number" so that this will happen. 3. Add equations* 4. Find the value of one variable by solving the resulting equation* 5. Substitute the value form Step 4 into either original equation to find the value of the other variable. 6. Solution is an ordered pair 7. Check the solution in both original equations	Solve $\begin{cases} x-2y=1 \\ 2x-4=6y \end{cases}$ 1. $x-2y=1$ $2x-6y=4$ 2.Multiply both sides of the first equation by -2 $-2x+4y=-2$ $2x-6y=4$ 3. $-2y=2$ 4. $y=-1$ 5. $x-2(-1)=1$ $x=-1$ 6. $(-1,-1)$ 7. $-2(-1)+4(-1)=-2$ $-2=-2\surd$ $2(-1)-4=6(-1)$ $-6=-6\surd$

*If all variables are eliminated & the remaining equation is true (e.g. 5 = 5), then the lines are identical
If all variables are eliminated & the remaining equation is false (e.g. 5 = 4), then the lines are parallel

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Solving Word Problems

	1 Variable, 1 Equation Method	2 Variables, 2 Equations Method
① **UNDERSTAND THE PROBLEM** ▪ As you use information, ~~cross~~ it out or underline it.	In a recent election for mayor 800 people voted. Mr. Smith received three times as many votes as Mr. Jones. How many votes did each candidate receive?	
② **DEFINE VARIABLES** ▪ Create "Let" statement(s) ▪ The variables are usually what the problem is asking you to solve for	▪ *Name what x is* (Can only be one thing. When in doubt, choose the smaller thing) ▪ *Define everything else in terms of x* Let x = Number of votes Mr. J $3x$ = Number of votes Mr. S	Let x = Number of votes Mr. S y = Number of votes Mr. J
③ **WRITE THE EQUATION(S)** ▪ You need as many equations as you have variables	$x + 3x = 800$	▪ *Usually each sentence is an equation* $x + y = 800$ $x = 3y$
④ **SOLVE THE EQUATION(S)**	$4x = 800$ $x = 200$	$(3y) + y = 800$ (Substitution) $4y = 800$ $y = 200$
⑤ **ANSWER THE QUESTION** ▪ Answer must include units!	▪ *Go back to your "Let" statement* 200 = Number of votes Mr. J 600 = Number of votes Mr. S	▪ *Go back to your "Let" statement* 200 = Number of votes Mr. J ▪ *Go back to your "Equations" & solve for remaining variable* $x + (200) = 800$ $x = 600$ 600 = Number of votes Mr. S
⑥ **CHECK** ▪ Plug answers into equation(s)	$(200) + 3(200) = 800$ $800 = 800\surd$	$(600) + (200) = 800$ $800 = 800\surd$ $(600) = 3(200)$ $600 = 600\surd$

MA091 Beginning Algebra Summary

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Polynomial Definitions

Polynomial	▪ A SUM OF TERMS WHICH CONTAINS ONLY WHOLE NUMBER EXPONENTS AND NO VARIABLE IN THE DENOMINATOR. ▪ Refers to an expression; can have polynomial equations	▹ $x^2 + 2x + 1$
Degree of a Polynomial	▪ Express polynomial in simplified form. Sum the powers of each variable in the terms. The degree of a polynomial is the highest degree of any of its terms ▪ Determines number of x-intercepts	▹ $2x^5y - 4x^3y^3$ & $4y^5 + 3y^6$ are both 6th-degree
Names for polynomials according to terms	▪ MONOMIAL- 1 term ▪ BINOMIAL- 2 terms ▪ TRINOMIAL- 3 terms	▹ $3x$; $3(x + 1)$ ▹ $3x + 3$ ▹ $x^2 + 2x + 1$
Names for polynomials according to degree	▪ LINEAR – degree 1 (1st power of a variable) ▪ QUADRATIC – degree 2 ▪ CUBIC – degree 3 ▪ QUARTIC – degree 4	▹ $3x$ ▹ $3x^2$ ▹ $3x^3$ ▹ $3x^4$

Polynomial Operations

Multiplication (Multiply each term of the first polynomial by each term of the second polynomial, and then combine like terms)	Horizontal Method ▪ Can be used for any size polynomials	Ex: $(x-2)(x^2+5x-1)$ $= x(x^2+5x-1) - 2(x^2+5x-1)$ $= x \bullet x^2 + x \bullet 5x + x(-1) + (-2)x^2 + (-2)5x + (-2)(-1)$ $= x^3 + 5x^2 - x - 2x^2 - 10x + 2$ $= x^3 + 3x^2 - 11x + 2$
	Vertical Method ▪ Can be used for any size polynomials. ▪ Similar to multiplying two numbers together	Ex: $(x-2)(x^2+5x-1)$ $x^2 \quad 5x \quad -1$ $x \quad -2$ $-2x^2 \quad -10x \quad 2$ $x^3 \quad 5x^2 \quad -x$ $x^3 \quad +3x^2 \quad -11x \quad +2$
	FOIL Method ▪ May only be used when multiplying two binomials. First terms, Outer terms, Inner terms, Last terms	Ex: $(x-2)(x-3)$ F O I L $= x \bullet x + x(-3) + (-2)x + (-2)(-3)$ $= x^2 - 3x - 2x + 6$ $= x^2 - 5x + 6$
Division	Dividing a Polynomial by a Monomial $\frac{a+b+c}{d} = \frac{a}{d} + \frac{b}{d} + \frac{c}{d}$	Ex: $\frac{2x+2}{4} = \frac{2x}{4} + \frac{2}{4} = \frac{x}{2} + \frac{1}{2}$ Ex: factor & canel $\frac{2x+2}{4} = \frac{2(x+1)}{4} = \frac{x+1}{2} = \frac{x}{2} + \frac{1}{2}$

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Factor Out the GCF

Factoring	▪ Writing an expression as a product ▪ Numbers can be written as a product of primes. Polynomials can be written as a product of *prime polynomials* ▪ Useful to simplify rational expressions and to solve equations ▪ The opposite of multiplying	▹ Factored $2(x+2)$ ▹ Not factored $2x+4$ $2 \bullet x + 2 \bullet 2$ factoring ▹ $2x+4=2(x+2)$ multiplying
GCF (Greatest Common Factor) of a List of Integers	1. Write each number as a product of prime numbers 2. Identify the common prime factors 3. The PRODUCT OF ALL COMMON PRIME FACTORS found in Step 2 is the GCF. If there are no common prime factors, the GCF is 1	▹ Find the GCF of 18 & 30 $18 = 2 \bullet 3 \bullet 3$ $30 = 2 \bullet 3 \bullet 5$ $\text{GCF} = 2 \bullet 3$ $= 6$
GCF of a List of Variables	▪ The variables raised to the smallest power in the list	▹ Find the GCF of x & x^2 $\text{GCF} = x$
GCF of a List of Terms	▪ The product of the GCF of the numerical coefficients and the GCF of the variable factors	▹ Find the GCF of $18x$& $30x^2$ $\text{GCF} = 6x$
Factor by taking out the GCF	1. Find the GCF of all terms 2. Write the polynomial as a product by factoring out the GCF 3. Apply the distributive property 4. Check by multiplying	▹ $-2x^2+6x^3$ $=(-2x^2)\bullet 1+(-2x^2)\bullet(-3x)$ $=-2x^2(1-3x)$ $=-2x^2+6x^3$ √ ▹ $-x^2+1$ $=(-1)\bullet(x^2)+(-1)\bullet(-1)$ $=-1(x^2-1)$ $=-x^2+1$ √

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Factor 4 Term Expressions	
a + b + c + d = (? + ?)(? + ?)	
▪ FACTOR BY GROUPING 1. Arrange terms so the 1st 2 terms have a common factor and the last 2 have a common factor 2. For each pair of terms, factor out the pair's GCF 3. If there is now a common binomial factor, factor it out 4. If there is no common binomial factor, begin again, rearranging the terms differently. If no rearrangement leads to a common binomial factor, the polynomial cannot be factored.	▹ Factor $10ax-6xy-9y+15a$ 1. $\underline{10ax+15a}\ \underline{-6xy-9y}$ 2. $5a(2x+3)-3y(2x+3)$ $(2x+3)(5a-3y)$

Factor Trinomials: Leading Coefficient of 1	
$x^2+bx+c=(x+?)(x+?)$	
▪ TRIAL & ERROR Product is c $(x + \text{one number})(x + \text{other number})$ Sum is b 1. Place x as the first term in each binomial, then determine whether addition or subtraction should follow the variable $x^2+bx+c=(x+d)(x+e)$ $x^2-bx+c=(x-d)(x-e)$ $x^2\pm bx-c=(x+d)(x-e)$ 2. Find all possible pairs of integers whose product is c 3. For each pair, test whether the sum is b 4. Check with FOIL √	Ex: Factor $x^2+7x+10$ 1. $(x+\quad)(x+\quad)$ 2. $2\bullet 5=10$ $1\bullet 10=10$ 3. $2+5=7$ - YES $1+10=11$ - NO $(x+5)(x+2)$ 4. $x^2+7x+10$ √

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Factor Any Trinomial

$ax^2 + bx + c = (?x + ?)(?x + ?)$

▪ **METHOD 1** (trial & error) 1. Try various combinations of factors of ax^2 and c until a middle term of bx is obtained when checking. 2. Check with FOIL √	Ex: Factor: $3x^2 + 14x - 5$ Product is $3x^2$ Product is -5 $(3x-1)(x+5)$ $15x - x = 14x$ (correct middle term)
▪ **METHOD 2** (*ac,* factor by grouping) 1. Identify a, b, and c 2. Find 2 "magic numbers" whose product is ac and whose sum is b. *Factor trees can be very useful if you are having trouble finding the magic numbers* (See MA090) 3. Rewrite bx, using the "magic numbers" found in Step 2 4. Factor by grouping 5. Check with FOIL √	Ex: Factor: $3x^2 + 14x - 5$ 1. $a = 3$ $b = 14$ $c = -5$ 2. $ac = (3) \bullet (-5) = -15$ $b = 14$ $(15) \bullet (-1) = -15$ √ $(15) + (-1) = 14$ √ "magic numbers" 15, –1 3. $3x^2 + 15x - x - 5$ 4. $3x(x + 5) - 1(x + 5)$ $(x + 5)(3x - 1)$
▪ **METHOD 3** (quadratic formula) 1. Use the quadratic formula to find the x values (or roots) 2. For each answer in step 1., rewrite the equation so that it is equal to zero 3. Multiply the two expressions from step 2, and that is the expression in factored form. 4. Check with FOIL √	Ex: Factor: $3x^2 + 14x - 5$ 1. $a = 3$ $b = 14$ $c = -5$ $x = \frac{-14 \pm \sqrt{14^2 - 4(3)(-5)}}{6}$ $x = \frac{1}{3}, -5$ 2. $x = \frac{1}{3}$ $x - \frac{1}{3} = 0$ $3x - 1 = 0$ $x = -5$ $x + 5 = 0$ 3. $(3x-1)(x+5)$

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Easy to Factor Polynomial Types

Perfect Square Trinomials $a^2 \pm 2ab + b^2$	▪ Factors into perfect squares (a binomial squared) $a^2+2ab+b^2=(a+b)^2$ $a^2-2ab+b^2=(a-b)^2$	▹ $9x^2+24x+16=(3x)^2+2(3x)(4)+(4)^2$ $=(3x+4)^2$ $(a=3x, b=4)$ ▹ $9x^2-24x+16=(3x)^2-2(3x)(4)+(4)^2$ $=(3x-4)^2$ $(a=3x, b=4)$
Difference of Squares a^2-b^2	▪ Factors into the sum & difference of two terms $a^2-b^2=(a+b)(a-b)$	▹ $x^2-1=(x)^2-(1)^2$ $(a=x, b=1)$ $=(x+1)(x-1)$
Sum of Squares a^2+b^2	▪ Does not factor $a^2+b^2=$ Prime	▹ x^2+1 is prime
Difference of Cubes a^3-b^3 (MA103)	▪ $a^3-b^3=(a-b)(a^2+ab+b^2)$	▹ $8x^3-27=(2x)^3-(3)^3$ $(a=2x, b=3)$ $=(2x-3)(4x^2+6x+9)$
Sum of Cubes a^3+b^3 (MA103)	▪ $a^3+b^3=(a+b)(a^2-ab+b^2)$	▹ $8x^3+27=(2x)^3+(3)^3$ $(a=2x, b=3)$ $=(2x+3)(4x^2-6x+9)$
Prime Polynomials (P)	▪ Can not be factored	▹ x^2+3x+1 is prime ▹ x^2-3 is prime

Factor Any Polynomial

1. Are the variable terms in descending order of degree with the constant term last? If not, put them in descending order.	Ex. $-32+2x^4$ $=2x^4-32$
2. Are there any common factors? If so, see *Factor Out the GCF (p.10)*	$=2(x^4-16)$
3. How many terms? ▪ 2 TERMS – see if one of the following can be applied • *Difference of Squares (p.13)* • *Sum of Cubes (p.13)* • *Difference of Cubes (p.13)* ▪ 3 TERMS – try one of the following • *Perfect Square Trinomial (p.13)* • *Factor Trinomials: Leading Coefficient of 1 (p.11)* • *Factoring Any Trinomial (p.12)* ▪ 4 TERMS – try *Factor by Grouping (p.11)*	$=2(x^2+4)(x^2-4)$
4. If both steps 2 & 3 produced no results, the polynomial is prime. You're done ☺ Skip steps 5 & 6	
5. See if any factors can be factored further	$=2(x^2+4)(x+2)(x-2)$
6. Check by multiplying	$=\left[2(x^2+4)\right]\left[(x+2)(x-2)\right]$ $=(2x^2+8)(x^2-4)$ $=2x^4-32$ √

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Quadratic Methods		
Standard Form	▪ $ax^2 + bx + c = 0$	▹ $x^2 - 3x + 2 = 0$
Solutions	▪ Has n solutions, where n is the highest exponent	▹ $x^3 - 3x^2 + 2x = 0$ (has 3 solutions)
Zero Factor Property	1. If a product is 0, then a factor is 0	▹ $xy = 0$ (either x or y must be zero)
Solve by Factoring	1. Write the equation in standard form (equal 0) 2. Factor 3. Set each factor containing a variable equal to zero 4. Solve the resulting equations	▹ $x^2 - 3x + 2 = 0$ 1. $x(x - 1)(x - 2) = 0$ 2. $x = 0, x - 1 = 0, x - 2 = 0$ 3. $x = 0, 1, 2$
Solve with the Quadratic Equation	• To solve a quadratic equation that is difficult or impossible to factor • Steps 1. Write the values for $a, b, \ \& \ c$ $(\textit{if a term does not exist, the coefficient is } 0)$ 2. Plug values into the quadratic equation below: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ 3. Simplify solutions and usually leave them in their most exact form ($\textit{Negative radicand means no real solutions}$)	Ex Radicand is a perfect square $x^2 - 3x + 2 = 0$ $a = 1, b = (-3), c = 2$ $x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(2)}}{2(1)} = \frac{3 \pm \sqrt{1}}{2}$ $= 2, 1$ Ex Radicand breaks into "perfect square" and "leftovers" $x^2 + 6x - 1 = 0$ $a = 1, b = 6, c = (-1)$ $x = \frac{-(6) \pm \sqrt{(6)^2 - 4(1)(-1)}}{2(1)}$ $= \frac{-6 \pm \sqrt{40}}{2} = \frac{-6 \pm 2\sqrt{10}}{2}$ $= \frac{-6}{2} \pm \frac{2\sqrt{10}}{2} = -3 \pm \sqrt{10}$ Ex Radicand is just "leftovers" $4x^2 - x - 1 = 0$ $a = 4, b = (-1), c = (-1)$ $x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(4)(-1)}}{2(4)}$ $= \frac{1 \pm \sqrt{17}}{8}$

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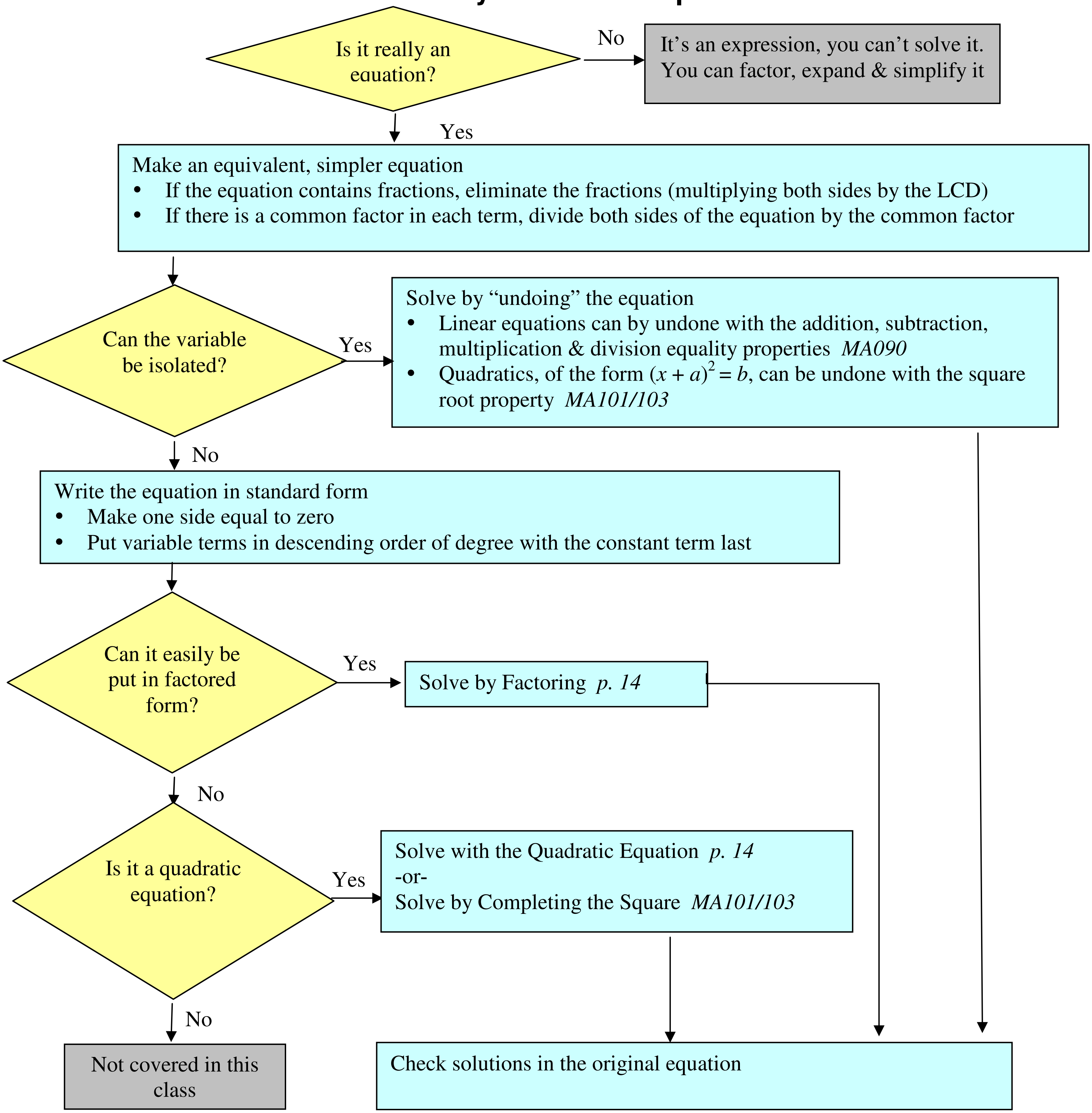
Solve Any 1 Variable Equation



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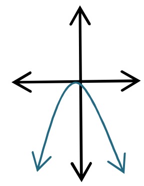
Graph Quadratic Equations in 2 Variables

Standard Form	$y = ax^2 + bx + c$ ▪ a, b, and c are real constants	▹ $y = x^2 - 9x + 20$
Solution	▪ A parabola	
Simple Form	$y = ax^2$ ▪ Vertex (high/low point) is (0,0) ▪ Line of symmetry is $x = 0$ ▪ The parabola opens up if $a > 0$, down if $a < 0$	▹ $y = -4x^2$
Graph	1. Plot y value at vertex 2. Plot y value one unit to the left of the vertex 3. Plot y value one unit to the right of the vertex	▹ $y = -4x^2$ x | y: 0 | 0; –1 | –4; 1 | –4 

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Exponents & Powers

Exponential Notation ▪ Shorthand for repeated multiplication	$base \rightarrow x^a \leftarrow exponent$	$2^3 = 2\bullet2\bullet2 = 8$
Multiplying Common Bases ▪ Add powers	$x^a \bullet x^b = x^{a+b}$	$2^2 \bullet 2^3 = 2^{2+3} = 2^5 = 32$ $(3x^2)(2y)(4x) = 24x^3y$
Powers of Powers ▪ Multiply powers	$\left(x^a\right)^b = x^{a\bullet b}$	$\left(2^3\right)^2 = 2^{3\bullet2} = 2^6 = 64$
Product Rule ▪ Raise each factor to the power	$(xy)^a = x^a \bullet y^a$ $(x^m y^n)^a = x^{ma} \bullet y^{na}$	$\left(2x^3\right)^2 = 2^2x^6 = 4x^6$
Quotient Rule ▪ Raise the dividend and divisor to the power	$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$	$\left(\frac{2}{z}\right)^2 = \frac{2^2}{z^2} = \frac{4}{z^2}$
Dividing Common Bases ▪ Subtract powers	$\frac{x^m}{x^n} = x^{m-n}$	$\frac{3^5}{3^3} = 3^{5-3} = 3^2 = 9$
Zero Power ▪ One	$x^0 = 1$, when $x \neq 0$	$6x^0 = (6)(1) = 6$
Negative Power ▪ Reciprocal of positive power ▪ *When simplifying, eliminate negative powers*	$x^{-n} = \frac{1}{x^n}$	$2^3 \bullet 2^{-3} = \frac{2^3}{1} \bullet \frac{1}{2^3} = 1$

Scientific Notation

Scientific Notation	▪ Shorthand for writing very small and large numbers ▪ $a \bullet 10^r$, where $1 \le a < 10$ & r is an integer	$(1.2\bullet10^2)\times(1.2\bullet10^3)$ $= 1.44\bullet10^5$	
Standard Form	▪ Long way of writing numbers	$120\times1200 = 144000$	
Standard Form to Scientific Notation	1. Move the decimal point in the original number to the left or right so that there is **one** digit before the decimal point 2. Count the number of decimal places the decimal point is moved in step 1 ▪ If the original number is 10 or greater, the count is positive ▪ If the original number is less than 1, the count is negative 3. Multiply the new number is step 1 by 10 raised to an exponent equal to the count found in step 2	510. 5.10 +2 $5.1\bullet10^2$	.051 05.1 –2 $5.1\bullet10^{-2}$
Scientific Notation to Standard Form	1. Multiply numbers together	$5.1\bullet10^2$ $= 5.1\bullet100$ $= 510$	$5.1\bullet10^{-2}$ $= 5.1\bullet\frac{1}{100}$ $= .051$

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Roots		
Roots	▪ Undoes raising to powers $\sqrt[2]{81}=9$ because $9^2=81$ index ↘ $\sqrt[2]{81}$ ↖ radical ↖ radicand	▷ $\sqrt{81}=\sqrt[2]{81}=9$ (The square root of 81 is 9) ▷ $\sqrt{27}=\sqrt[3]{27}=3$ (The cube root of 27 is 3)
Computation	▪ If *n* IS AN EVEN POSITIVE INTEGER, then $\sqrt[n]{a^n}=\|a\|$ The radical $\sqrt{\ }$ represents only the non-negative square root of *a*. The $-\sqrt{\ }$ represents the negative square root of *a*. ▪ IF *n* IS AN ODD POSTIVIE INTEGER, then $\sqrt[n]{a^n}=a$	▷ $\sqrt{9}=\sqrt{3^2}=\|3\|=3$ ▷ $\sqrt{(-3)^2}=\|-3\|=3$ ▷ $\sqrt{(x+1)^2}=\|x+1\|$ ▷ $\sqrt{-9}$ "Not a real number" ▷ $-\sqrt{9}=-\sqrt{3^2}=-\|3\|=-3$ ▷ $\sqrt{.09}=\|.3\|=.3$ $(.3\bullet.3=.09)$ ▷ $\sqrt{3}\approx\|1.73\|\approx1.73$ (approximately) ▷ $\sqrt[3]{27}=\sqrt[3]{3^3}=3$ ▷ $\sqrt[3]{-27}=\sqrt[3]{(-3)^3}=-3$

Notation:
Radical vs.
Rational Exponent
	▪ The root of a number can be expressed with a radical or a rational exponent
	▪ Rational exponents
	  ▪ The numerator indicates the power to which the base is raised.
	  ▪ The denominator indicates the index of the radical
		▷ $\sqrt[3]{27}=(27)^{1/3}$
		▷ $\sqrt[3]{27^2}=\left(\sqrt[3]{27}\right)^2=27^{2/3}=\left(27^{1/3}\right)^2$
		▷ $\frac{1}{\sqrt[3]{27^2}}=\left(\frac{1}{\sqrt[3]{27}}\right)^2=27^{-2/3}=\left(27^{1/3}\right)^{-2}$
		Note, it's usually easier to compute the root before the power

Operations	▪ Roots are powers with fractional exponents, thus power rules apply.	▷ $\sqrt[3]{-8x^3}=(-8x^3)^{1/3}$ $=(-8)^{1/3}(x^3)^{1/3}=-2x$
Product Rule	$\sqrt[n]{a}\sqrt[n]{b}=\sqrt[n]{ab}$	▷ $\sqrt{6}\sqrt{7}=\sqrt{42}$
Quotient Rule	$\sqrt[n]{\frac{a}{b}}=\frac{\sqrt[n]{a}}{\sqrt[n]{b}}$, provided $\sqrt[n]{b}\neq0$	▷ $\sqrt{\frac{1}{25}}=\frac{\sqrt{1}}{\sqrt{25}}=\frac{1}{5}$
Simplifying Expressions	1. Separate radicand into "perfect squares" and "leftovers" 2. Compute "perfect squares" 3. "Leftovers" stay inside the radical so the answer will be exact, not rounded	▷ Just perfect squares... $\sqrt{36x^2}=6x$ ▷ Prefect squares & leftovers... $\sqrt{32x^3}=\sqrt{16x^2}\sqrt{2x}=4x\sqrt{2x}$ ▷ Just leftovers... $\sqrt{33x}=\sqrt{33x}$

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Rational Expressions

Rational Numbers	▪ Can be expressed as quotient of integers (fraction) where the denominator ≠ 0 ▪ All integers are rational ▪ All "terminating" decimals are rational	▹ 0 = 0/1 ▹ 4 = 4/1 ▹ 4.25 = 17/4
Irrational Numbers	▪ Cannot be expressed as a quotient of integers. Is a non-terminating decimal	▹ $\pi = 3.141592654...$ ▹ $\sqrt{2} = 1.414213562...$
Rational Expression	1. An expression that can be written in the form $\frac{P}{Q}$, where P and Q are polynomials 2. Denominator ≠ 0	▹ $\frac{x}{x+6}$, Find real numbers for which this expression is undefined: $x + 6 = 0$; $x = -6$
Simplifying Rational Expressions (factor)	1. Completely factor the numerator and denominator 2. Cancel factors which appear in both the numerator and denominator	▹ Simplify $\frac{4x+20}{x^2-25}$ $= \frac{4(x+5)}{(x+5)(x-5)}$ $= \frac{4}{x-5}$
Multiplying/ Dividing Rational Expressions (multiply across)	1. If it's a division problem, change it to a multiplication problem 2. Factor & simplify 3. Multiply numerators and multiply denominators 4. Write the product in simplest form	▹ Simplify $\frac{x}{x+6} \cdot \frac{3}{x} = \frac{3x}{x(x+6)}$ $= \frac{3}{x+6}$
Adding/ Subtracting Rational Expressions (get common denominator)	1. Factor & simplify each term 2. Find the LCD ❖ The LCD is the product of all unique factors, each raised to a power equal to the greatest number of times that it appears in any one factored denominator 3. Rewrite each rational expression as an equivalent expression whose denominator is the LCD 4. Add or subtract numerators and place the sum or difference over the common denominator 5. Write the result in simplest form	▹ Simplify $\frac{x}{x+6} + \frac{3}{6}$ LCD $= 6(x+6)$ $= \frac{?}{6(x+6)} + \frac{?}{6(x+6)}$ $= \frac{(6)x}{(6)(x+6)} + \frac{3(x+6)}{6(x+6)}$ $= \frac{6x}{6(x+6)} + \frac{3x+18}{6(x+6)}$ $= \frac{9x+18}{6(x+6)}$ $= \frac{9^3(x+2)}{6^2(x+6)}$ $= \frac{3x+6}{2(x+6)}$

Solving Rational Equations		
Solving by Eliminating the Denominator	1. Factor & simplify each term 2. Multiply both sides *(all terms)* by the LCD 3. Remove any grouping symbols 4. Solve 5. Check answer in **original equation**. If it makes any of the denominators equal to 0 (undefined), it is not a solution	Solve $\frac{x}{x+6}+\frac{3}{x}=1$ LCD $= x(x+6)$ $[x(x+6)]\left[\frac{x}{x+6}+\frac{3}{x}\right]=[x(x+6)]1$ $\left[\frac{x\cancel{(x+6)}}{1}\right]\left[\frac{x}{\cancel{x+6}}\right]+\left[\frac{\cancel{x}(x+6)}{1}\right]\left[\frac{3}{\cancel{x}}\right]=[x(x+6)]1$ $x(x)+3(x+6)=1(x^2+6x)$ $x^2+3x+18=x^2+6x$ $x=6$ Check $\frac{(6)}{(6)+6}+\frac{3}{(6)}=1$ $\frac{1}{2}+\frac{1}{2}=1\surd$
Solving Proportions with the Cross Product $\frac{a}{b}=\frac{c}{d}$	*If your rational equation is a proportion, it's easier to use this shortcut* 1. Set the product of the diagonals equal to each other 2. Solve 3. Check	Solve $\frac{3}{4}=\frac{x}{x-1}$ $3(x-1)=4x$ $3x-3=4x$ $x=-3$ Check $\frac{3}{4}=\frac{(-3)}{(-3)-1}\surd$

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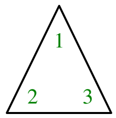
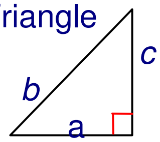
Formulas

Geometric	Triangle 	▪ SUM OF ANGLES: Angle 1 + Angle 2 + Angle 3 = 180^{o}
	Right Triangle 	▪ PYTHAGOREAN THEOREM: $a^2 + b^2 = c^2$ (a = leg, b = leg, c = hypotenuse) ~The hypotenuse is the side opposite the right angle. It is always the longest side.
Other	Distance →	▪ DISTANCE: $d = rt$ (r = rate, t = time)

Dictionary of Terms

Real Numbers	▪ Points on a number line ▪ Whole numbers, integers, rational and irrational numbers	▹ $7, -7, \frac{7}{2}, \pi$
Positive Infinity (Infinity)	▪ An unimaginably large positive number. (If you keep going to the right on a number line, you will never get there)	⟷ ∞ or $+\infty$
Negative Infinity	▪ An unimaginably small negative number. (If you keep going to the left on a number line, you will never get there)	⟷ $-\infty$