Polynomial division:

Question: When might we want to divide one polynomial by another?

For example, dividing \( f(x) = x^4 + 6x^3 + 11x^2 + 12x + 4 \) by \( g(x) = x^2 + 3x + 2 \)

Answer: One reason might be to find the roots (values of \( x \) that make \( f(x) = 0 \)).

You could use a process called synthetic division to find the roots of a polynomial.

However, if you find synthetic division confusing and/or difficult to remember, then you can also use simple division.

Example: Suppose we want to find the roots of the polynomial

\[ f(x) = 6x^4 + 17x^3 - 29x^2 - 2x + 8 \]

The roots of this polynomial are given by \( \pm \frac{P}{Q} \), where \( P \) is a factor of 8 and \( Q \) is a factor of 6.

The factors of 8 are: 1 x 8, 2 x 4

The factors of 6 are: 1 x 6, 2 x 3

So the possible values of \( \frac{P}{Q} \) are \( \pm 8, \pm 4, \pm 2, \pm 1, \pm \frac{1}{2}, \pm \frac{8}{3}, \pm \frac{4}{3}, \pm \frac{2}{3}, \pm \frac{1}{3}, \pm \frac{1}{6} \)

To see if \( x = 1 \) is a root, then we need to divide \( f(x) \) by \( (x - 1) \):

Start by writing the problem like one in long division:

\[
\begin{array}{c}
6x^4 + 17x^3 - 29x^2 - 2x + 8 \\
\hline
x - 1
\end{array}
\]

Divide \( 6x^4 \) by \( x \) to get \( 6x^3 \). Write that above the horizontal line:

\[
\begin{array}{c}
6x^3 \\
\hline
x - 1 \quad 6x^4 + 17x^3 - 29x^2 - 2x + 8
\end{array}
\]

Now multiply \( (x - 1) \) by \( 6x^3 \), write below \( f(x) \), and subtract:

\[
\begin{array}{c}
6x^3 \\
\hline
x - 1 \quad 6x^4 + 17x^3 - 29x^2 - 2x + 8
\end{array}
\]

\[
\begin{array}{c}
\hline
6x^4 - 6x^3 \\
\hline
23x^3 - 29x^2
\end{array}
\]

After subtracting, bring down the next term: \(-29x^2\)
Now divide $23x^3$ by $x$ to get $23x^2$ and repeat:

\[
\begin{array}{c|ccccc}
& 6x^3 & + & 23x^2 \\
\hline
x - 1 & 6x^4 & + & 17x^3 & - & 29x^2 & - 2x & + 8 \\
6x^4 & - & 6x^3 & & & & & \\
\hline
& 23x^3 & - & 29x^2 \\
& 23x^3 & - & 23x^2 \\
\hline
& - 6x^2 & - 2x \\
& - 6x^2 & + & 6x \\
\hline
& - 8x + 8 \\
\end{array}
\]

And finally:

\[
\begin{array}{c|ccccc}
& 6x^3 & + & 23x^2 & - 6x & - 8 \\
\hline
x - 1 & 6x^4 & + & 17x^3 & - & 29x^2 & - 2x & + 8 \\
6x^4 & - & 6x^3 & & & & & \\
\hline
& 23x^3 & - & 29x^2 \\
& 23x^3 & - & 23x^2 \\
\hline
& - 6x^2 & - 2x \\
& - 6x^2 & + & 6x \\
\hline
& - 8x + 8 \\
& - 8x + 8 \\
\hline
& 0 \\
\end{array}
\]

So the answer is:

\[
6x^4 + 17x^3 - 29x^2 - 2x + 8 = (x - 1) (6x^3 + 23x^2 - 6x - 8)
\]

Exercises:

Divide the following:

1. $(x^3 - 1)$ by $(x - 1)$
2. $(x^3 - 4x^2 - 20x + 48)$ by $(x - 2)$
3. $(x^3 + 6x^2 + 11x + 6)$ by $(x + 3)$
4. $(x^4 + 8x^3 + 24x^2 + 32x + 16)$ by $(x^2 + 4x + 4)$

Answers: 1. $x^2 + x + 1$ 2. $x^2 - 2x + 2$ 3. $x^2 - 2x + 1$ 4. $x + 2$