Logarithm Basics:

For every logarithmic equation of the form \( N = \log_b(X) \) there is an exactly equivalent exponential equation: \( X = b^N \)

\( \log_b(X) \) is “the logarithm to the base b of X” and is the number to which you raise b in order to get X

So we can think of logarithms as powers of the base, b.

“Taking the logarithm of” and “raising to a power” are inverse operations, i.e., each “undoes” the other:

\( N = \log_b(b^N) \quad \text{and} \quad X = b^{\log_b(X)} \)

For every rule for exponents there is a corresponding rule for logarithms:

<table>
<thead>
<tr>
<th>Exponents</th>
<th>Logarithms</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b^0 = 1 )</td>
<td>( \log_b(1) = 0 )</td>
</tr>
<tr>
<td>( b^1 = b )</td>
<td>( \log_b(b) = 1 )</td>
</tr>
<tr>
<td>( b^{-1} = 1/b )</td>
<td>( \log_b(1/b) = -1 )</td>
</tr>
<tr>
<td>( b^N \cdot b^M = b^{N+M} )</td>
<td>( \log_b(X \cdot Y) = \log_b(X) + \log_b(Y) )</td>
</tr>
<tr>
<td>( b^N / b^M = b^{N-M} )</td>
<td>( \log_b(X/Y) = \log_b(X) - \log_b(Y) )</td>
</tr>
<tr>
<td>( (b^N)^M = b^{N \cdot M} )</td>
<td>( \log_b(X^N) = N \cdot \log_b(X) )</td>
</tr>
</tbody>
</table>

By convention: \( \log(X) = \log_{10}(X) \) Rule for changing base (e.g., to base 10):

\[ \ln(X) = \log_e(X) \]

\[ \log_b(X) = \frac{\log(X)}{\log(b)} \]

where \( e \approx 2.718 \)
Logarithms of integers up to 10:

<table>
<thead>
<tr>
<th>N</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(N)</td>
<td>0.30</td>
<td>0.48</td>
<td>0.60</td>
<td>0.70</td>
<td>0.78</td>
<td>0.85</td>
<td>0.90</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Exercises:

For each of 1 – 8, match the expression or equation with an equivalent expression or equation in a) – h).

1. \( \log_5 25 \)
2. \( 2^5 = X \)
3. \( \log_5 5 \)
4. \( \log_2 1 \)
5. \( \log_5 5^x \)
6. \( \log_X 27 = 5 \)
7. \( 8 = 2^X \)
8. \( X^{-2} = 5 \)

a) \( 1 \)   b) \( X \)   c) \( X^5 = 27 \)   d) \( \log_2 X = 5 \)   e) \( \log_2 8 = X \)
 f) \( \log_X 5 = -2 \)
g) \( 2 \)   h) \( 0 \)

Simplify:

9. \( \log_{10} 1000 \)
10. \( \log_2 16 \)
11. \( \log_8 1 \)
12. \( \log_5 9^5 \)
13. \( \log_{10} 0.01 \)
14. \( \log_2 8 \)
15. \( \log_2 8 \)
16. \( \log_4 \frac{1}{4} \)
17. \( \log_7 \frac{1}{49} \)
18. \( \log_5 125 \)
19. \( \log_9 3 \)
20. \( \log_9 27 \)
21. \( \log_{27} 9 \)
22. \( \log_{16} 64 \)
23. \( 6^{\log_6 13} \)
24. \( \log_{1/4} \frac{1}{64} \)
25. \( \log_{81} 3 \times \log_3 81 \)
26. \( \log_{10}(\log_4(\log_3 81)) \)

Solve:

27. \( |\log_3 X| = 2 \)
28. \( \log_4(3X - 2) = 2 \)
29. \( \log_8 (2X + 1) = -1 \)
30. \( \log_{10}(X^2 + 21X) = 2 \)