Simplifying Algebraic Expressions to Standard Polynomial Form:

How might we simplify an expression like

$$2(x - 1)(x - 2)(x + 3) - (x^2 - 4)(x + 2) + 5x^3 + (x^2 - 1)(x^2 + 1) - 10(x + 1)$$

to standard polynomial form?

One approach* would be to break the problem into the following steps:

1. Multiply each term separately, keeping the parentheses:

   \[
   \begin{align*}
   &2(x - 1)(x - 2)(x + 3) - (x^2 - 4)(x + 2) + 5x^3 + (x^2 - 1)(x^2 + 1) - 10(x + 1) \\
   &= 2(x^2 - 2x - x + 2)(x + 3) - (x^3 + 2x^2 - 4x - 8) + (x^4 + x^2 - x^2 - 1) - (10x^2 + 10x)
   \end{align*}
   \]

2. Combine like terms:

   \[
   2(x^2 - 3x + 2)(x + 3) - (x^3 + 2x^2 - 4x - 8) + (x^4 - 1) - (10x^2 + 10x)
   \]

3. If necessary, repeat steps 1 and 2, until all terms have been multiplied:

   Multiply each term by each term

   \[
   \begin{align*}
   &2(x^2 - 3x + 2)(x + 3) - (x^3 + 2x^2 - 4x - 8) + (x^4 - 1) - (10x^2 + 10x) \\
   &= 2(x^3 - 3x^2 + 2x + 3x^2 - 9x + 6) - (2x^3 - 14x + 12) - (x^3 + 2x^2 - 4x - 8) + (x^4 - 1) - (10x^2 + 10x)
   \end{align*}
   \]

3. When all terms have been multiplied, distribute the negative (−) signs:

   \[
   (2x^3 - 14x + 12) + (-x^3 - 2x^2 + 4x + 8) + (x^4 - 1) + (-10x^2 - 10x)
   \]

*If you already have an approach that works for you, that’s great! Remember that there’s almost always more than one right way to approach a math problem.
4. Now drop the parentheses:

\[2x^3 - 14x + 12 - x^3 - 2x^2 + 4x + 8 + x^4 - 1 - 10x^2 - 10x\]

5. Find the term with the highest power of \(x\). Find all terms with this power and combine.

Move the result to the beginning:

\[x^4 + 2x^3 - 14x + 12 - x^3 - 2x^2 + 4x + 8 - 1 - 10x^2 - 10x\]

6. Repeat for the next highest power of \(x\):

\[x^4 + x^3 - 14x + 12 - 2x^2 + 4x + 8 - 1 - 10x^2 - 10x\]

7. Continue until all like terms have been combined:

\[x^4 + x^3 - 12x^2 - 14x + 12 + 4x + 8 - 1 - 10x\]

\[x^4 + x^3 - 12x^2 - 20x + 12 + 8 - 1\]

\[x^4 + x^3 - 12x^2 - 20x + 19\]

And we’re done!

Exercises:

Simplify the following expressions:

1. \((x + 1)(x + 4) - (2x^3 - 1) + (x^2 - 1)(x - 2)\)
2. \(6x^4 - 4(x - 1)(x^3 + 1) + 5(x^2 + 1) - x^3 - 10\)

Answers: 1. 

\[-x^4 - 2x^3 + 5x^2 + 7x + 4x + \frac{x}{2} - \frac{x}{2} - x - 1\]