Mathematics Foundations for Success in Introductory Statistics

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The Charles A. Dana Center invited the authors to present their views on the foundational skills that students need to be successful in a college-level statistics course. The authors provide a set of mathematics foundations that would prepare students for Introductory Statistics. This resource is offered to faculty who are reviewing placement and prerequisite requirements in their own departments.

For more information about the Dana Center's position on the mathematics foundations for Introductory Statistics courses, see the Call to Action at https://dcmathpathways.org/resources/call-action-expand-access-statistics

Many colleges and universities are now exploring multiple pathways to a credit-bearing, college-level mathematics course. Because the required mathematics course for a wide variety of majors—such as nursing, criminal justice, social work, psychology and kinesiology—is statistics, much attention is now focused on providing a productive pathway to statistics.

In order to place students appropriately and in order to design an efficient and effective pathway to statistics for students who may need additional support, it is important to think carefully about the mathematical foundations for success in the introductory statistics course. While these foundations include topics typically taught in courses up to and including beginning and intermediate algebra, there are topics in beginning and intermediate algebra that are not necessary for success in an introductory statistics course.

"The authors have compiled a dependable reference that identifies the mathematical knowledge and skills employed in college-level Introductory Statistics courses. This document includes an emphasis on the development of the critical thinking ability our students will need as the changing landscape of the statistics profession continues to impact the requirements for statistical literacy."

Professor Mary DeHart
Chair, AMATYC/ASA Joint Committee
American Mathematical Association of Two-Year Colleges (AMATYC)/American Statistical Association (ASA)

Statement endorsed by the AMATYC Executive Board

This paper describes the topics and concepts that are considered necessary mathematics preparation for success in statistics. In the table that follows, the mathematics foundations for statistics have been grouped into the following general categories: numbers and the number line, operations on numbers, sets, equations and inequalities, graphing points and lines in two dimensions, and reading tables and graphs and approximating areas. The first column of the table lists mathematical foundations, and the second column provides examples of content topics in the introductory statistics course that are dependent on mastery of the associated mathematical foundation.

Numbers and the Number Line	
Students need to be able to . . .	**In order to . . .**
Plot points and intervals on the number line	Make and interpret dotplots
Represent an inequality as an interval on the number line	Calculate probabilities for continuous random variables, understand and interpret confidence interval estimates
Find the distance between two points on the number line	Calculate deviations from the mean and calculate z-scores
Round decimals	Calculate numerical summary statistics, test statistics, and confidence intervals
Order decimal numbers	Calculate medians and quartiles, and compare *P*-values to a significance level
Convert between fractions, decimals, and percents	Calculate and interpret probabilities, calculate margin of error and confidence intervals, interpret confidence levels and Type I and Type II error probabilities

Operations on Numbers	
Students need to be able to . . .	**In order to . . .**
Perform signed number arithmetic	Calculate residuals, z-scores, numerical summary statistics, test statistics, and confidence interval estimates
Calculate powers of a number (using technology)	Calculate the variance and standard deviation of a sample and the value of a chi-square statistic
Calculate the square root of a number (using technology)	Calculate standard deviation and standard error
Use summation notation	Calculate an expected value, the sample mean and standard deviation, the correlation coefficient, the value of the chi-square statistic, and regression coefficients
Understand order of operations in expressions and formulas	Calculate numerical summary statistics, test statistics, and confidence interval estimates

Sets	
Students need to be able to . . .	**In order to . . .**
Understand Venn diagrams	Understand probability rules and calculations
Use set notation	Define sample spaces and events
Find the complement of a set	Define events and calculate their probabilities
Find the union and the intersection of two sets	Define events and calculate their probabilities

Equations and Inequalities	
Students need to be able to . . .	**In order to . . .**
Evaluate algebraic expressions	Calculate numerical summary statistics, test statistics, confidence intervals, z-scores and regression coefficients
Solve a linear equation in one variable	Find percentiles for a normal distribution

Graphing Points and Lines in Two Dimensions	
Students need to be able to . . .	**In order to . . .**
Plot an ordered pair (x, y) in a rectangular coordinate system	Create scatterplots and residual plots
Understand slope as the change in y associated with a 1-unit change in x	Understand and interpret regression coefficients in a data context
Given the equation of a line, draw the graph of the line	Graph the regression line
Use the equation of a line to find the y-value associated with a given x-value	Use the regression line to make predictions
Find the vertical distance between a point and a line	Calculate residuals

Reading Tables and Graphs and Approximating Areas	
Students need to be able to . . .	**In order to . . .**
Extract information from tables and graphs	Interpret graphical displays of data
Given the total area under a curve or a histogram, approximate the area of a shaded region	Approximate probabilities and P-values and understand graphical displays of data

Because data analysis is conducted in context, the practice of statistics involves reading, problem solving, and writing in context. For this reason, experience in applying mathematical tools in real-world settings and interpreting results in context should also be a part of any course that is designed to support students in an introductory statistics course.

The focus of a contemporary introductory statistics course is primarily on developing statistical thinking and conceptual understanding, with much less emphasis on hand calculations than was the case in the past. Because the use of statistical software and/or graphing calculators is an integral part of the course, previous work with a graphing calculator or with computer applications is desirable, although not essential.

Students with mastery of the mathematics foundations described here would be adequately prepared for a college-level introductory statistics course. For students who require additional support, a pathway that focuses on this content—whether preceding the introductory statistics course or as a co-requisite support course coordinated with the order of topics in the introductory statistics course—would provide an efficient pathway to statistics.