Statistical Thinking and GAISE

Allan J. Rossman
Department of Statistics
Cal Poly – San Luis Obispo
Email: arossman@calpoly.edu
Blog: https://askgoodquestions.blog

GAISE

- Guidelines for Assessment and Instruction in Statistics Education
- Recommendations for teaching introductory statistics at college level
  - Comparable guidelines at PreK-12 level
- Developed by American Statistical Association
  - Originally in 2005, revised in 2016
- www.amstat.org/education/gaise

Poll question #1

- How would you describe your comfort level with teaching statistics?
  - 1: Terrified, feel like I know nothing!
  - 2: Some comfort, but lots to learn
  - 3: Fairly comfortable, but still a ways to go
  - 4: Quite comfortable, always looking for new ideas
  - 5: I know everything, just here to stir up trouble!

Poll question #2

- How familiar are you with GAISE recommendations (before two minutes ago)?
  - 1: Never heard of GAISE
  - 2: Have heard of GAISE but that's it
  - 3: Have perused GAISE report or attended a presentation about GAISE
  - 4: Have thought a good bit about how GAISE relates to my teaching of statistics
  - 5: Could recite GAISE recommendations verbatim

GAISE recommendations

1. Teach statistical thinking.
2. Focus on conceptual understanding.
3. Integrate real data with a context and purpose.
4. Foster active learning.
5. Use technology to explore concepts and analyze data.
6. Use assessments to improve and evaluate student learning.

1. Teach statistical thinking

- Example 1: Sex discrimination?

	Men	Women
Accepted	533	113
Denied	665	336
Total	1198	449

- Men: 533/1198 ≈ .445 were accepted
- Women: 113/449 ≈ .252 were accepted
- Does this provide evidence of discrimination against women?

1. Teach statistical thinking

	Men		Women	
	Accepted	Denied	Accepted	Denied
Program A	511	314	89	19
Program F	22	351	24	317
Total	533	665	113	336

- Program A
  - Men: 511/825 ≈ .619
  - Women: 89/108 ≈ .824
- Program F:
  - Men: 22/373 ≈ .059
  - Women: 24/341 ≈ .070

1. Teach statistical thinking

- What's surprising about these percentages?
  - Men had a much higher acceptance rate than women overall
  - But women had a higher acceptance rate than men in each program
- Explain how this happened, using the data given
  - Most men applied to the program with high acceptance rates (A)
  - Most women applied to the program with low acceptance rates (F)

1. Teach statistical thinking

- Example 2: Cancer pamphlets

Patients' reading levels	< 3	3	4	5	6	7	8	9	10	11	12	> 12	Total
Count (number of patients)	6	4	4	3	3	2	6	5	4	7	2	17	63

Pamphlets' readability level	6	7	8	9	10	11	12	13	14	15	16	Total
Count (number of pamphlets)	3	3	8	4	1	1	4	2	1	2	1	30

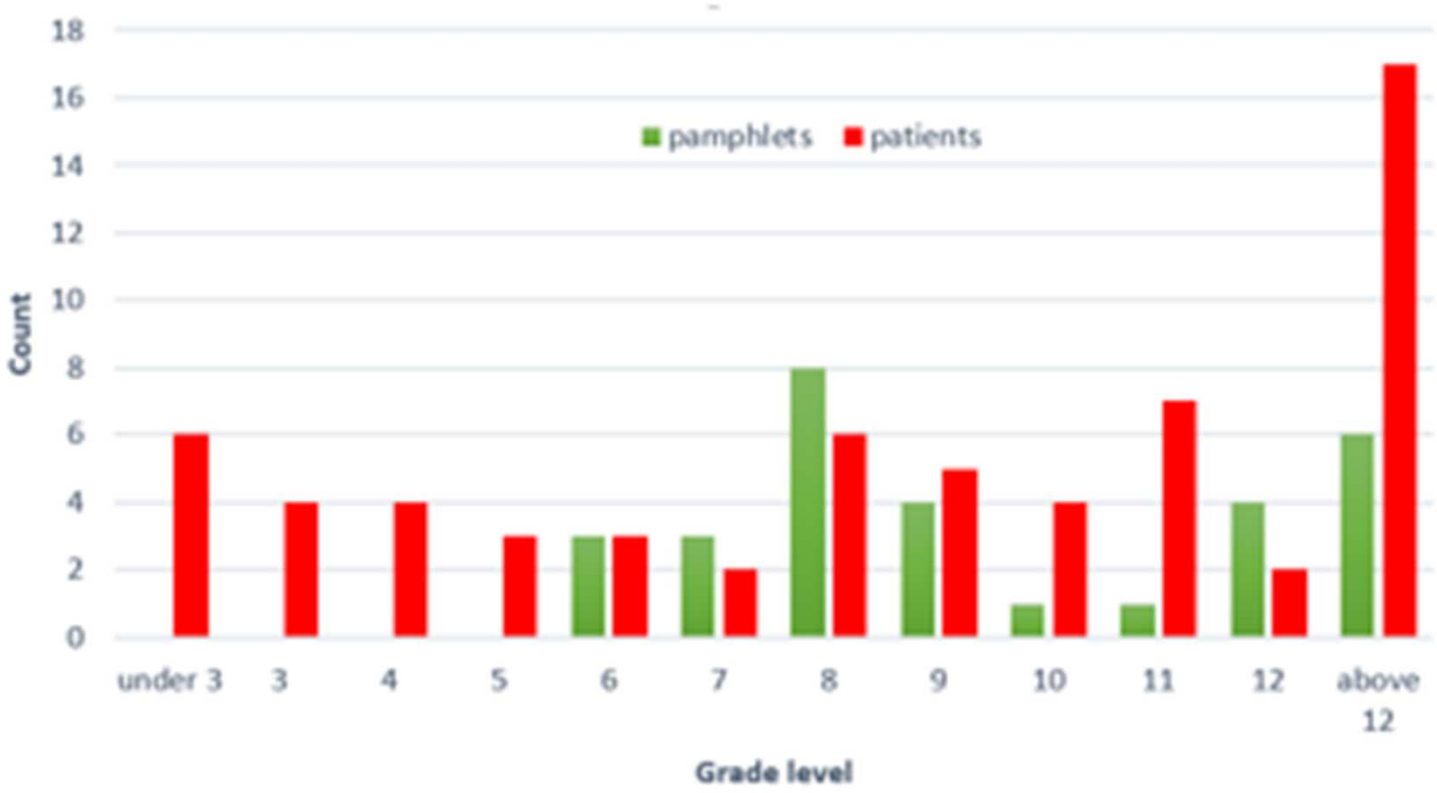
- Determine median for each distribution.
- Do medians reveal that pamphlets are written at appropriate levels to be read by patients?

1. Teach statistical thinking

- Median = 9th grade for both (patients, pamphlets)
- Does this mean that pamphlets' readability levels are well-matched to patients' reading levels?
- No! Need to compare *distributions*, not centers
  - Many patients (17/63 ≈ 27%) have reading level below that of simplest pamphlet

1. Teach statistical thinking

- Visual displays can be illuminating



1. Teach statistical thinking

- Example 3: Draft lottery
- On December 1, 1969, the United States implemented a lottery to draft young men into the military (https://www.youtube.com/watch?v=-p5X1FjyD_g)
- Each of the 366 birthdays of the year was assigned a draft number
  - Those with smaller draft numbers were drafted
- Find your birthday's draft number
  - Any 236s?

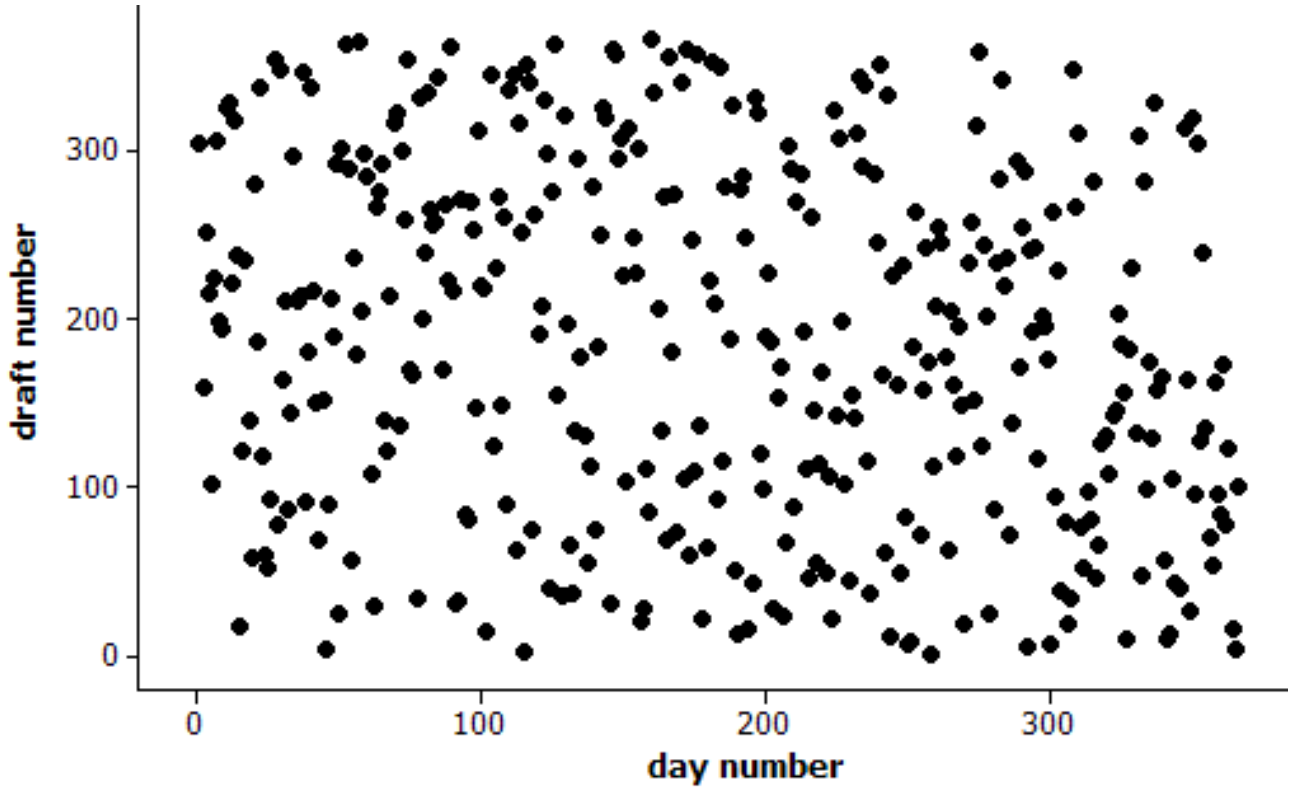
1. Teach statistical thinking

date	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1	305	86	108	32	330	249	93	111	225	359	19	129
2	159	144	29	271	298	228	350	45	161	125	34	328
3	251	297	267	83	40	301	115	261	49	244	348	157
4	215	210	275	81	276	20	279	145	232	202	266	165
5	101	214	293	269	364	28	188	54	82	24	310	56
6	224	347	139	253	155	110	327	114	6	87	76	10
7	306	91	122	147	35	85	50	168	8	234	51	12
8	199	181	213	312	321	366	13	48	184	283	97	105
9	194	338	317	219	197	335	277	106	263	342	80	43
10	325	216	323	218	65	206	284	21	71	220	282	41
11	329	150	136	14	37	134	248	324	158	237	46	39
12	221	68	300	346	133	272	15	142	242	72	66	314
13	318	152	259	124	295	69	42	307	175	138	126	163
14	238	4	354	231	178	356	331	198	1	294	127	26
15	17	89	169	273	130	180	322	102	113	171	131	320

1. Teach statistical thinking

date	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
16	121	212	166	148	55	274	120	44	207	254	107	96
17	235	189	33	260	112	73	98	154	255	288	143	304
18	140	292	332	90	278	341	190	141	246	5	146	128
19	58	25	200	336	75	104	227	311	177	241	203	240
20	280	302	239	345	183	360	187	344	63	192	185	135
21	186	363	334	62	250	60	27	291	204	243	156	70
22	337	290	265	316	326	247	153	339	160	117	9	53
23	118	57	256	252	319	109	172	116	119	201	182	162
24	59	236	258	2	31	358	23	36	195	196	230	95
25	52	179	343	351	361	137	67	286	149	176	132	84
26	92	365	170	340	357	22	303	245	18	7	309	173
27	355	205	268	74	296	64	289	352	233	264	47	78
28	77	299	223	262	308	222	88	167	257	94	281	123
29	349	285	362	191	226	353	270	61	151	229	99	16
30	164		217	208	103	209	287	333	315	38	174	3
31	211		30		313		193	11		79		100

1. Teach statistical thinking



- Any reason to doubt randomness?

1. Teach statistical thinking

- Calculate median draft number for each month
- Draft numbers sorted within each month from smallest to largest

rank	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1	17	4	29	2	31	20	13	11	1	5	9	3
2	52	25	30	14	35	22	15	21	6	7	19	10
3	58	57	33	32	37	28	23	36	8	24	34	12
4	59	68	108	62	40	60	27	44	18	38	46	16
5	77	86	122	74	55	64	42	45	49	72	47	26
6	92	89	136	81	65	69	50	48	63	79	51	39
7	101	91	139	83	75	73	67	54	71	87	66	41
8	118	144	166	90	103	85	88	61	82	94	76	43
9	121	150	169	124	112	104	93	102	113	117	80	53
10	140	152	170	147	130	109	98	106	119	125	97	56
11	159	179	200	148	133	110	115	111	149	138	99	70
12	164	181	213	191	155	134	120	114	151	171	107	78
13	186	189	217	208	178	137	153	116	158	176	126	84
14	194	205	223	218	183	180	172	141	160	192	127	95
15	199	210	239	219	197	206	187	142	161	196	131	96
16	211	212	256	231	226	209	188	145	175	201	132	100
17	215	214	258	252	250	222	190	154	177	202	143	105
18	221	216	259	253	276	228	193	167	184	220	146	123
19	224	236	265	260	278	247	227	168	195	229	156	128
20	235	285	267	262	295	249	248	198	204	234	174	129
21	238	290	268	269	296	272	270	245	207	237	182	135
22	251	292	275	271	298	274	277	261	225	241	185	157
23	280	297	293	273	308	301	279	286	232	243	203	162
24	305	299	300	312	313	335	284	291	233	244	230	163
25	306	302	317	316	319	341	287	307	242	254	266	165
26	318	338	323	336	321	353	289	311	246	264	281	173
27	325	347	332	340	326	356	303	324	255	283	282	240
28	329	363	334	345	330	358	322	333	257	288	309	304
29	337	365	343	346	357	360	327	339	263	294	310	314
30	349		354	351	361	366	331	344	315	342	348	320
31	355		362		364		350	352		359		328

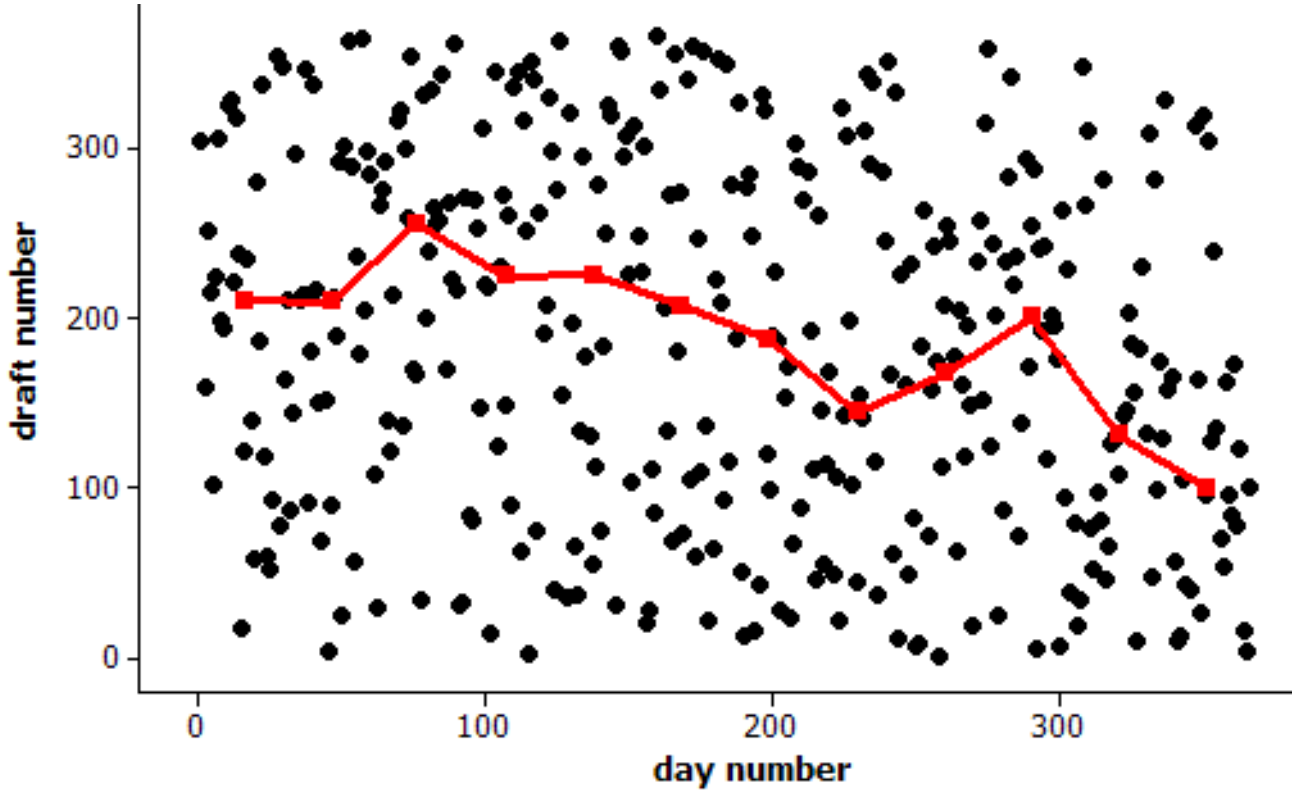
1. Teach statistical thinking

- Calculate median draft number for each month

Jan 211	Jul 188
Feb 210	Aug 145
Mar 256	Sep 168
Apr 225	Oct 201
May 226	Nov 131.5
Jun 207.5	Dec 100

- Do you see a pattern/trend?

1. Teach statistical thinking



- Any reason to doubt randomness?

1. Teach statistical thinking

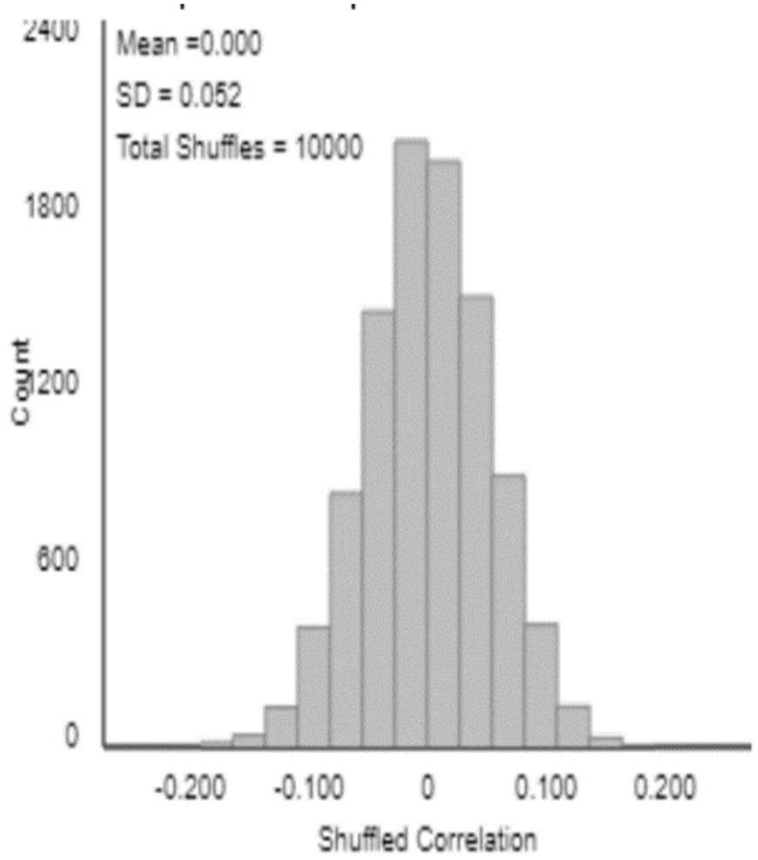
- How often would such an extreme outcome occur with a truly random lottery?

1. Teach statistical thinking

- How often would such an extreme outcome occur with a truly random lottery?
- What statistic might you use to determine "extreme"-ness?

1. Teach statistical thinking

- How often would such an extreme outcome occur with a truly random lottery?
- What statistic might you use to determine "extreme"-ness?
  - Correlation ≈ -0.226



2. Focus on conceptual understanding

- Example: Mean/average
- I suspect that when I moved from PA to CA, the average IQ dropped in both states!
  - Is this possible?
  - What would have to be true (in principle) for this to happen?
- Yes, if (my IQ > average IQ in PA)

and (my IQ < average IQ in CA)

2. Focus on conceptual understanding

- Example: Variability/SD

- Suppose that Abby records the ages of customers at The Avenue (on-campus snack bar) from 11am-2pm today, while Mary records ages of customers at McDonald's (near freeway).
- Who will have the larger standard deviation of customer ages: Abby or Mary? Explain.

2. Focus on conceptual understanding

- Example: Measuring distance as number of SDs away from *something*

- Exam scores have mean 70, SD 8
  - Arturo's score is 75
  - Bella's score is 1.5 SDs above Arturo's
- What is Bella's score on the exam?

2. Focus on conceptual understanding

- Example: Correlation

- Suppose that every student in our class scored 5 points lower on the second exam than on the first exam.
- What would be the value of the correlation coefficient between *exam1 score* and *exam2 score*?
  - Options: -5, -1, -0.5, 0, 0.5, 1, 5

3. Integrate real data

- Example: Facial prototyping

Do people tend to associate names with faces? (Lea, Thomas, Lamkin, & Bell, 2007)

Who is on the left: Bob or Tim?

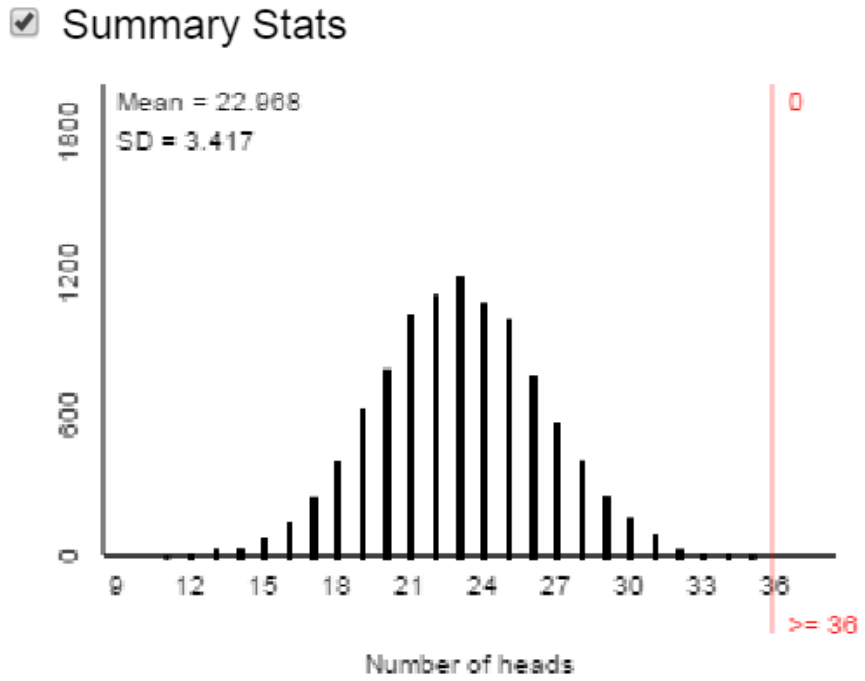
4. Foster active learning

Example: Facial prototyping (cont)

- 36 of 46 students put Tim on the left
  - What are two possible explanations for our observed sample result?
  - Which explanation can we investigate/model? How?
  - How often would such an extreme sample result occur by chance alone (if there were no facial prototyping)?
  - Have students flip coins to investigate

5. Use technology to explore concepts

- Facial prototyping: 10,000 simulated samples of 46 students



- Very strong evidence that people have a genuine tendency to put Tim on left

6. Use assessments to improve learning

- Almost every example

a) What are the observational units in this study?

b) What are the variables in this study? Which are categorical; which are numerical? Which is explanatory; which is response?

c) Did this study make use of random sampling, random assignment, both, or neither?

d) Is this an observational study or an experiment?

6. Use assessments to improve learning

- Almost every example

a) What are the observational units in this study?

b) What are the variables in this study? Which type is which variable? Which variable plays which role?

c) Did this study make use of random sampling, random assignment, both, or neither?

d) Is this an observational study or an experiment?

6. Use assessments to improve learning

- Example (probability):
  - A recent survey revealed that 48% of households in the U.S. have a pet dog, and 37% have a pet cat.
  - Does it follow that 48% + 37% = 85% of households in the U.S. have a pet dog *or* a pet cat? Explain why or why not.

6. Use assessments to improve learning

- Create an example of 10 exam scores such that the mean is at least 20 points less than the median and the IQR = 0.
- Create an example with 100 dog owners and 50 cat owners, such that the proportion of dog owners who have taken their pet to a vet in the past year is more than two times larger than the proportion of cat owners who have taken their pet to a vet in the past year.

6. Use assessments to improve learning

- Suppose that you want to test whether one-third of all adults in your county have a tattoo. Create an example of a sample with 500 people such that the two-sided p-value is less than .01.
- Create an example of 10 body temperatures such that the sample mean is 98.0 and a 95% CI for the population mean does not include the value 98.6.

6. Use assessments to improve learning

- Example (adapted from Jay Lehmann):

a) Which would be larger – the mean weight of 10 randomly selected people or the mean weight of 1000 randomly selected cats? Explain briefly.

b) Which would be larger – the standard deviation of the weights of 1000 randomly selected people or the standard deviation of the weights of 10 randomly selected cats? Explain briefly.

6. Use assessments to improve learning

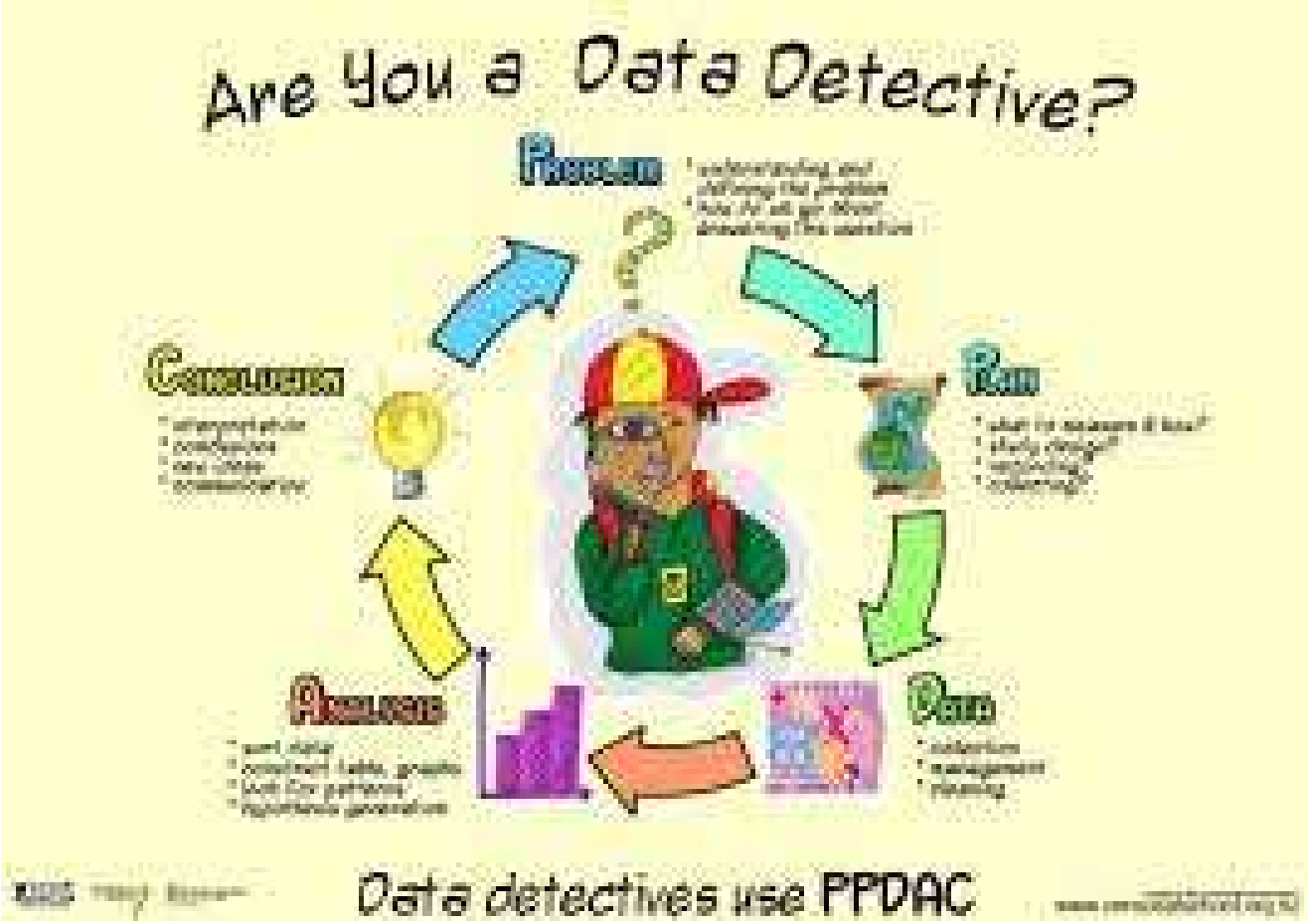
- Many of my students think that larger sample size produces smaller SD
- Do not realize that SD *of the sample mean* (or *sample proportion*) gets smaller as sample size increases
- Advice: When you talk about SD, always emphasize SD *of what*

My cats

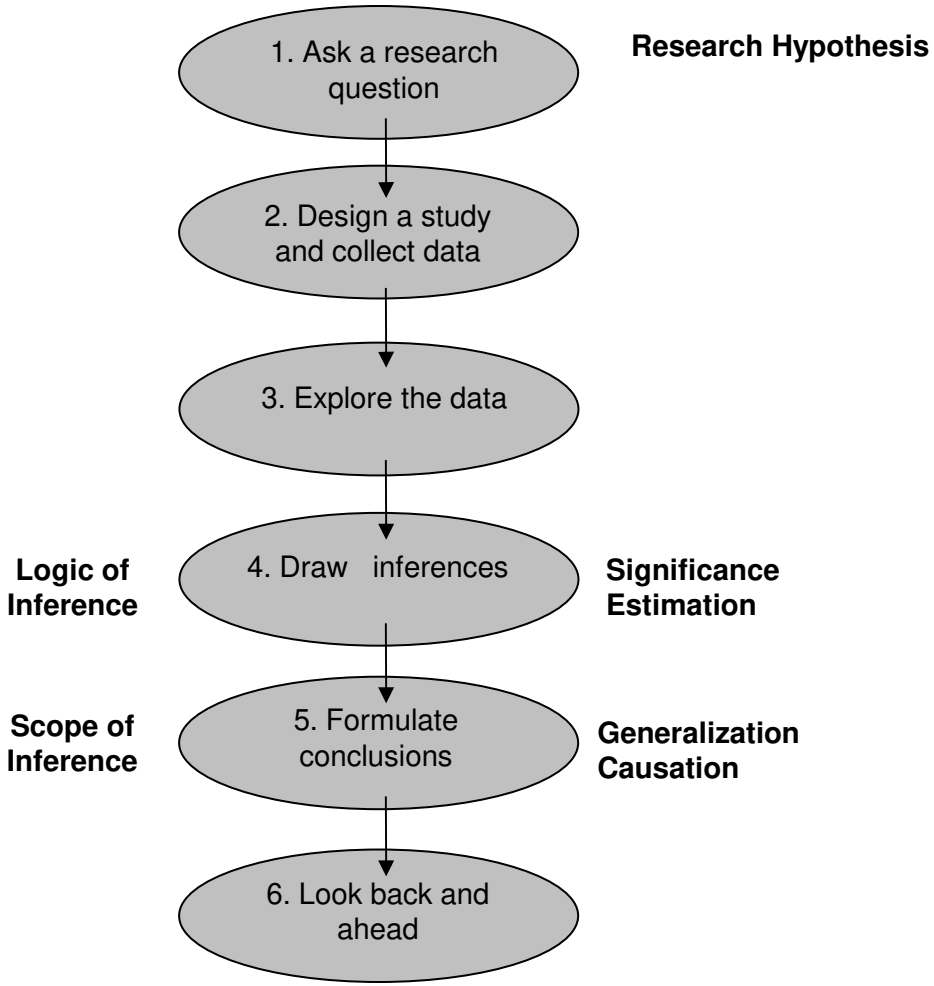
New emphases in GAISE revision

1. Teach statistical thinking
   a) Teach statistics as investigative process of problem-solving and decision-making
   b) Give students experience with multivariable thinking

1a. Investigative process



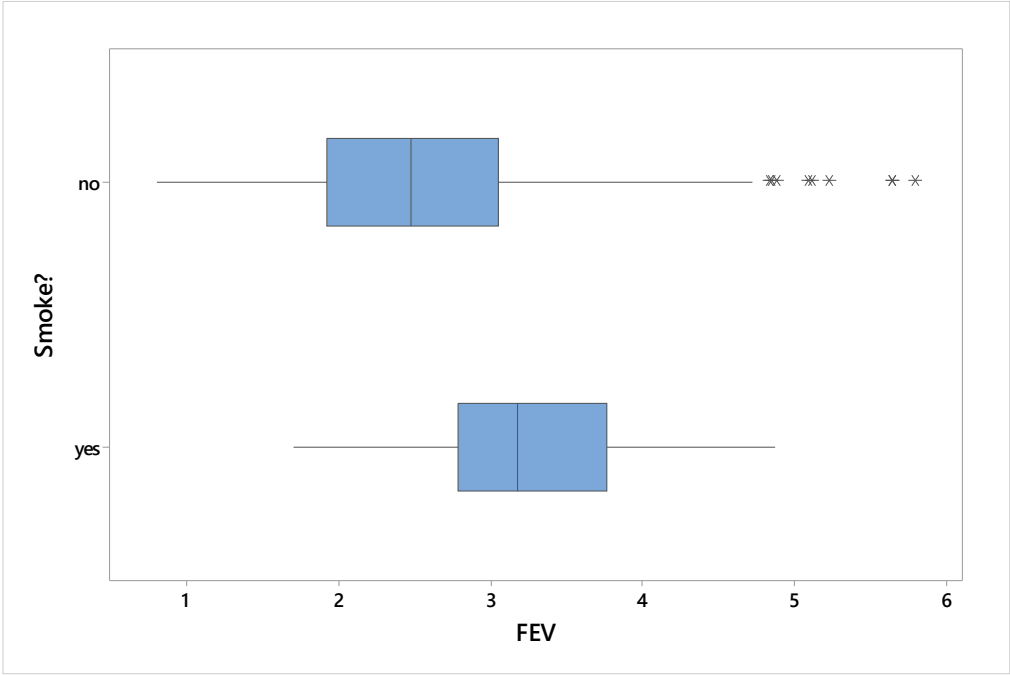
1a. Investigative process



1b. Multivariable thinking

- Example: Lung capacity and smoking

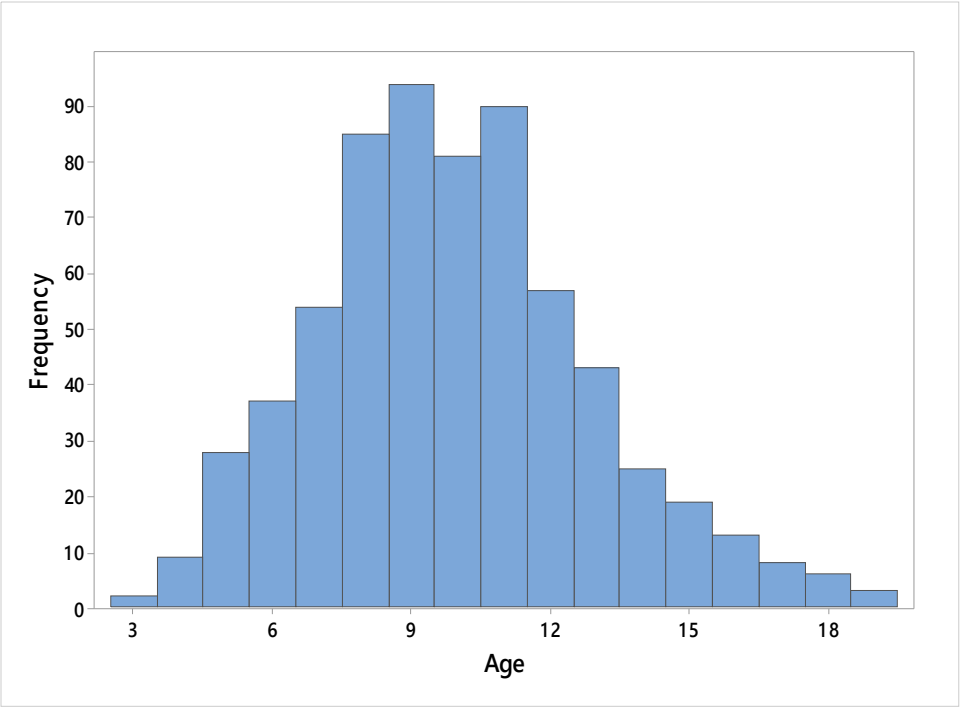
- $t = 7.15$
- p-value ≈ .0000



- What's going on here???

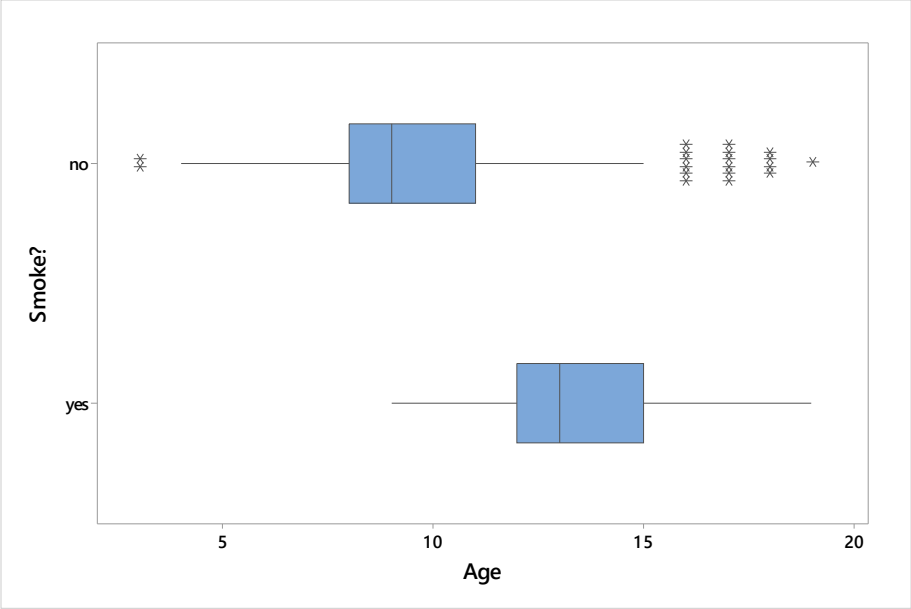
1b. Multivariable thinking (cont)

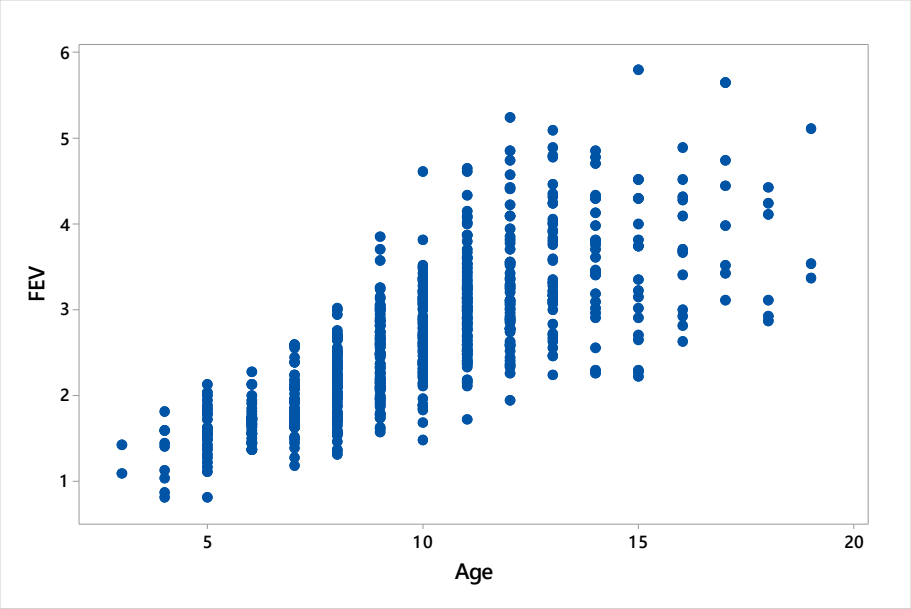
- Confounding variable
- These data are from children aged 3 – 19
- How does age explain why smokers have significantly larger lung capacities than non-smokers?



1b. Multivariable thinking (cont)

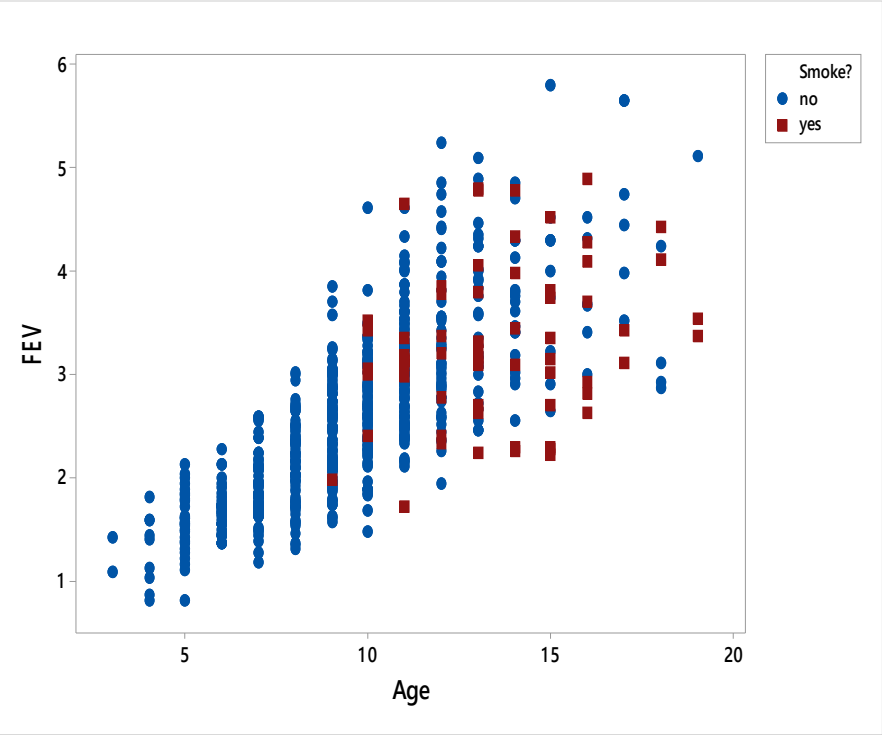
- Age is associated with both smoking status and lung capacity





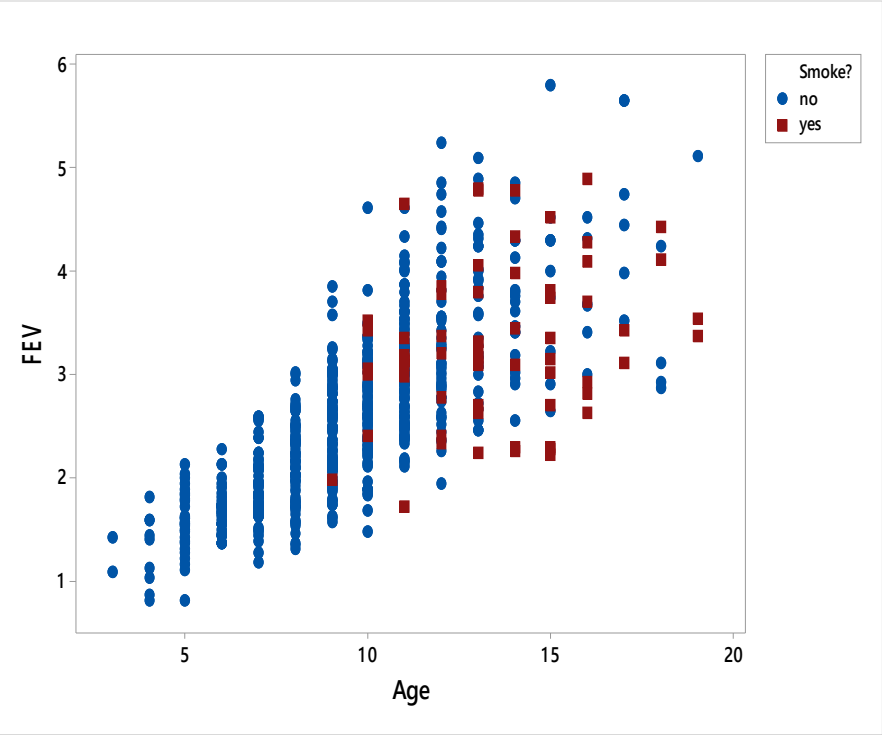
1b. Multivariable thinking (cont)

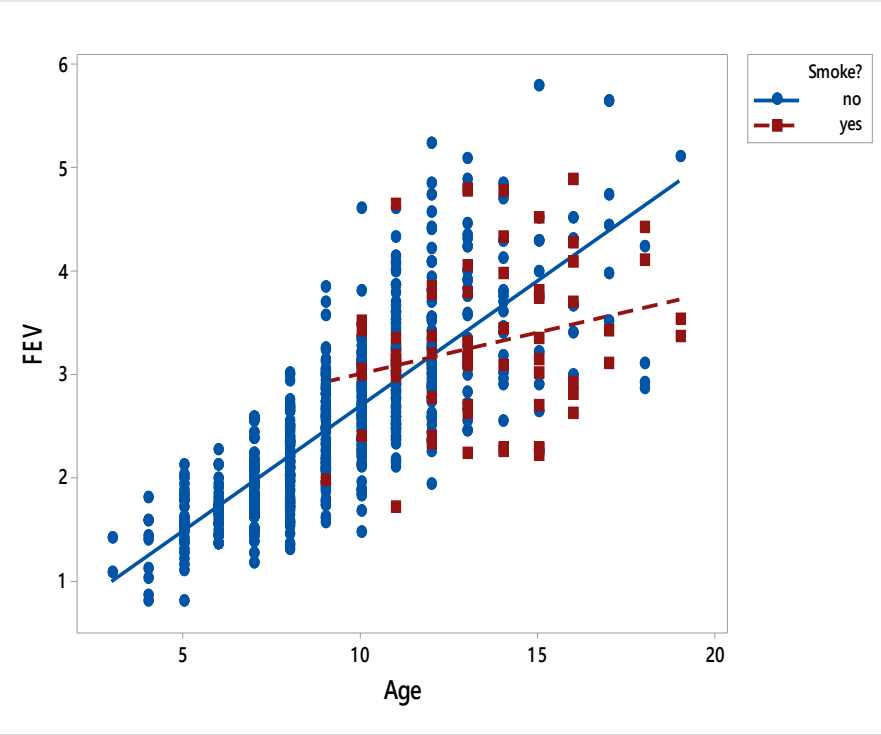
- Let's look at all three variables together



1b. Multivariable thinking (cont)

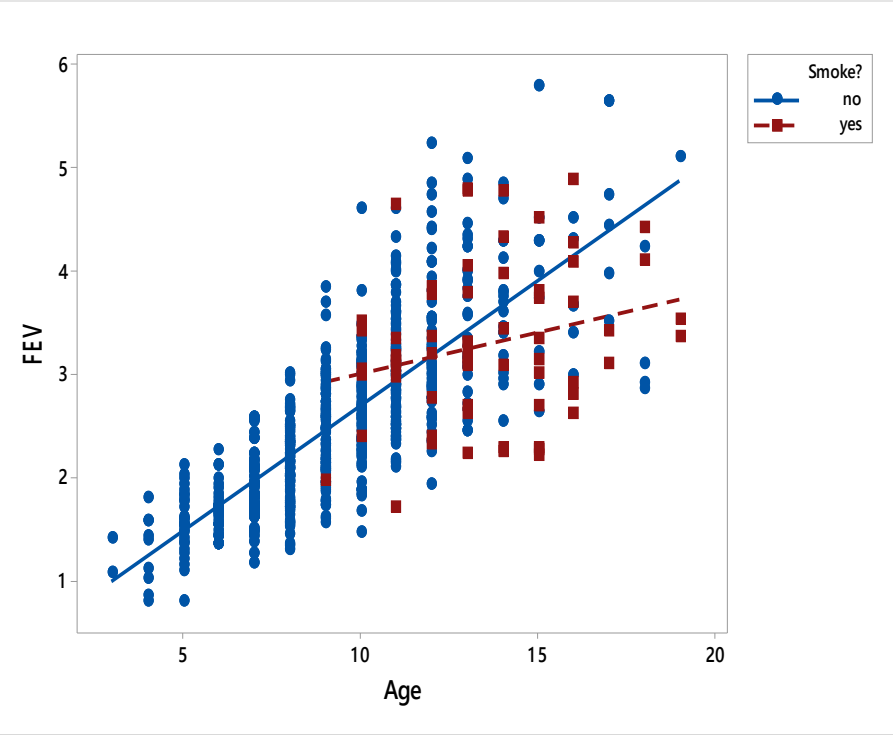
- Let's look at all three variables together





1b. Multivariable thinking (cont)

- After controlling for age, smokers have smaller lung capacity than non-smokers (12 or older)
- Rate of increase in lung capacity per year of age is smaller for smokers than for non-smokers



GAISE: Goals for introductory students

1. Become ***critical consumers***.
2. Be able to apply ***investigative process***.
3. Produce and interpret results of ***graphical displays*** and ***numerical summaries***.
4. Recognize and explain fundamental role of ***variability***.
5. Recognize and explain central role of ***randomness*** in designing studies and drawing conclusions.

GAISE: Goals for introductory students

6. Gain experience with ***statistical models***, including multivariable ones.
7. Demonstrate understanding of, and ability to apply, ***statistical inference*** in variety of settings.
8. Interpret and draw conclusions from standard output of ***statistical software***.
9. Demonstrate awareness of ***ethical issues*** associated with sound statistical practice.

My teaching philosophy

- What's the key to being a successful singer?
  - Sing Good Songs

My teaching philosophy

- What's the key to effective teaching?
  - Ask Good Questions

"Ask Good Questions" blog

- Ideas, examples, activities, assessments and advice for teaching introductory statistics
- Three goals
  - Practical, for direct use with students
  - Thought-provoking, for discussion with peers
  - Casual writing style, fun to read
- One post/essay per week
  - Published on Mondays at 11am ET, 8am PT
- askgoodquestions.blog

U.S. Conference on Teaching Statistics

Thanks very much!

- arossman@calpoly.edu
- www.amstat.org/education/gaise/
- www.rossmanchance.com/applets
- www.causeweb.org/uscots/uscots21/
- https://askgoodquestions.blog